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Nuclear & Particle Physics

for

CSIR-UGC-NET & GATE

First Edition	:	Sept. 2015
Second Edition	:	March 2016
Third Edition	:	April 2018

I study myself more than any other subject.
That is my metaphysics, that is my physics.

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Chapter 1

Basic Nuclear Properties

Introduction:

On the basis of α-ray scattering experiment, E. Rutherford described that inside the atom there is a very small nucleus in which all the positive charge and most of the mass is localized. In this chapter, we will study about nuclear charge, nuclear mass, size, shape, binding energy, angular momentum, magnetic moment, statistics and parity.

Structure of nuclei:

There are two types of particles inside the nucleus called nucleon.

(i) Proton : it has positive charge (Z or P) (ii) Neutron : it is neutral (N)

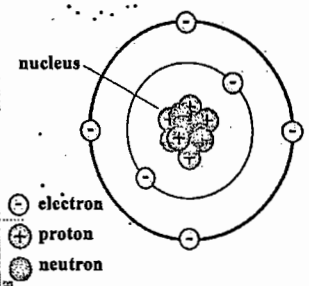

Figure 1.1: Atomic structure

Atomic number:

The number of protons inside the nucleus is called atomic number.

It is represented by 'Z'.

Mass number:

The total number of nucleons (proton + neutron) is called mass number. It is represented by 'A' and $A = Z + N$.

Symbolical representation of nucleus:

A nucleus is symbollically represented by ${}_{Z}X^{A}$ (or ${}^{A}X_{Z}$ or ${}_{Z}^{A}X$ or X_{Z}^{A}). For example, ${}_{8}O^{16}$, ${}_{2}He^{4}$.

Isotopes:

Nuclei with the same atomic number Z, but different mass number A are called isotopes.

For example, ${}_{1}H^{1}$, ${}_{1}H^{2}$, ${}_{1}H^{3}$ are isotopes of hydrogen.

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Isobars:

Nuclei with same mass number A but different atomic number Z are called isobars. For example, ${}_{8}O^{16}$ and ${}_{7}N^{16}$ are isobars.

Isotones:

Nuclei with same number of neutrons N are called isotones. For example, ${}_{7}N^{14}$ and ${}_{6}C^{13}$ are isotones.

Mirror nuclei:

A pair of isobaric nuclei are known as mirror nuclei where in the nucleus the proton number Z and the neutron number N are interchanged and differ by one unit is called Mirror nuclei, like ${}_{6}C^{11}$ and ${}_{5}B^{11}$ are mirror nuclei.

Binding energy of nucleus:

Inside the nucleus protons and neutrons are strongly bound. To separate nucleons from each other we need few MeV energy which is called the binding energy E_B of the nucleus. But the question arises that what is the source of this energy?

According to mass energy equivalence of special relativity, the energy equivalent corresponding to complete conversion of a mass m into energy is mc^2, where 'c' is the speed of light in free space. In forming a nucleus out of constituent particles, a fraction of the total mass of the constituents disappears and produce energy equal to E_B.

If ΔM be the amount of mass disappeared, then the energy, $E_B = \Delta Mc^2$

If M_P, M_N, M_e be the masses of proton, neutron and electron respectively

$$\Delta M = ZM_P + NM_N + ZM_e - M(A,Z)$$

where, $M(A,Z)$ is the mass of the atom of mass number A and atomic number Z.

$$E_B = \left[ZM_P + NM_N + ZM_e - M(A,Z)\right]c^2$$

$$= \left[ZM_P + NM_N + ZM_e - M_{nuc} - ZM_e\right]c^2$$

$$= \left[ZM_P + NM_N - M_{nuc}\right]c^2$$

$\Rightarrow$ $\boxed{E_B = \left[ZM_P + NM_N - M_{nuc}\right]c^2}$, where M_{nuc} is the mass of the nucleus.

Binding fraction curve:

Binding fraction f_B is the ratio of binding energy to mass number.

$\therefore$ $$f_B = \frac{E_B}{A} = \frac{\left[ZM_P + NM_N - M_{nuc}\right]c^2}{A}$$

$\boxed{f_B = \frac{\left[ZM_P + NM_N - M_{nuc}\right]c^2}{A}}$, f_B is also called binding energy per nucleon.

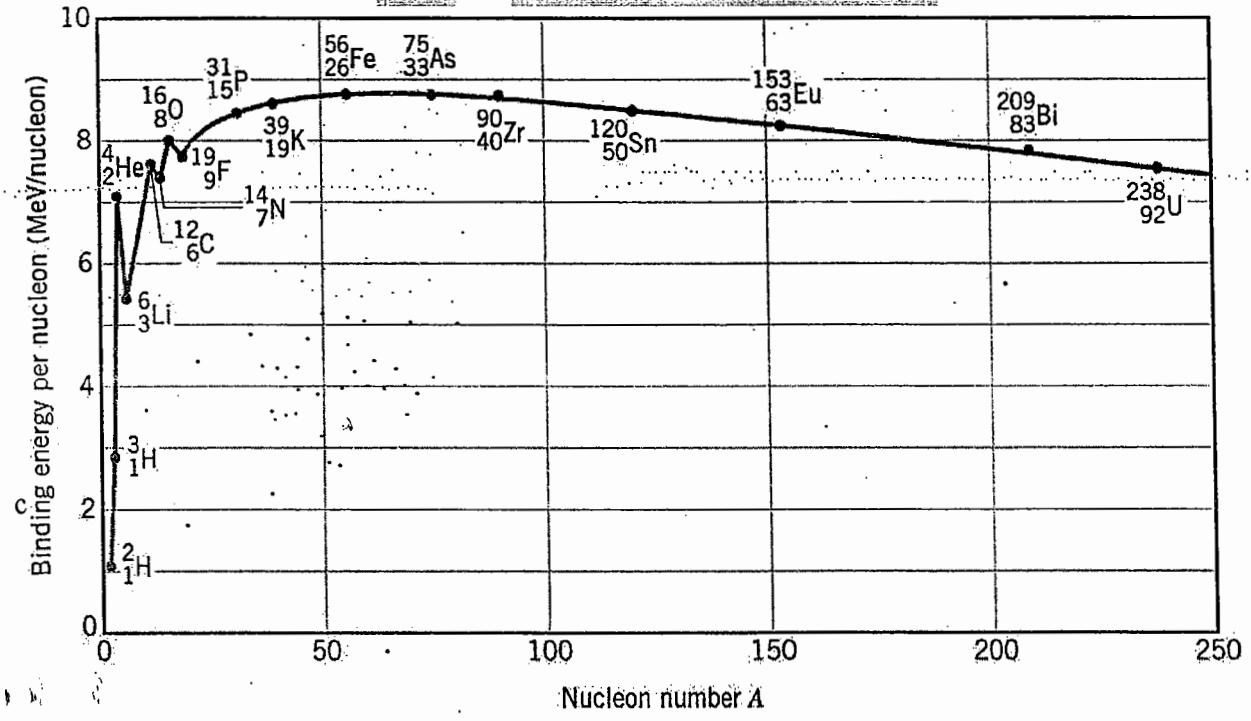

Figure 1.2: Plot of binding energy per nucleon (E_B/A) versus the nucleon number (A)

Observation from graph:

1. f_B is very small for very light nuclei and goes on increasing rapidly with increasing A and reaches a value ~ 8 MeV/nucleon for the mass number $A \sim 20$. Thereafter the rise of the curve is much slower, reaching a maximum value of 8.7 MeV per nucleon for $A = 56$. If A is increased still further, the curve decreases slowly.
2. The variation in f_B is very slight in the mass number range $20 < A < 180$ and in this region f_B may be considered to be virtually constant with a mean value ~ 8.5 MeV/nucleon.

3. For $A > 180$, that is, for heavy nuclei, the f_B-value decreases monotonically with increasing A and it is ~ 7.5 MeV/nucleon for the heaviest nuclei.
4. A rapid fluctuation in f_B is noted for very light nuclei with peaks in the curve of this region, corresponding to even-even nuclei, such as ^4He, ^8Be, ^{12}C, ^{16}O i.e., with mass number $A = 4n$, where $n = 1, 2, 3, \ldots\ldots$ Peaks in the curve are also seen at Z or N equal to 20, 28, 50, 82, 126. These are called magic numbers.

Mass defect:

The difference between the measured atomic mass $M(A,Z)$ and the mass number A of a nucleus is called the mass defect, $\Delta M'$

$$\therefore \quad \boxed{\Delta M' = M(A,Z) - A}$$

for $_8O^{16}$, $\Delta M' = 15.994915 - 16 = -0.005085$ a.m.u.

Packing fraction:

The packing fraction f is defined as the mass defect per nucleon in the nucleus.

$$f = \frac{\Delta M'}{A} \quad \frac{M(A,Z) - A}{A}$$

$$\Rightarrow \quad \boxed{M(A,Z) = A(1+f)}$$

Nuclear size:

From experimental result we can say that the mass density of the nucleus almost remain constant.

$$\rho_m = \frac{A}{V} = \text{constant}$$

$$\therefore \quad V \propto A$$

$$\Rightarrow \quad \frac{4}{3}\pi R^3 \propto A \Rightarrow R \propto A^{1/3}$$

$$\Rightarrow \quad \boxed{R = R_0 A^{1/3}}$$, where r_0 is a proportionality constant and its value lie in the range $(1.1 - 1.5) \times 10^{-15}$ m. R_0 is generally taken as 1.2×10^{-15} m.

Charge distribution of nucleus:

If $\rho(x,y,z)$ be the volume density of charge of the nucleus with its centre of mass at the origin so that $r^2 = x^2 + y^2 + z^2$. Also the charge density can be expressed as

$$\boxed{\rho(r) = \frac{\rho_0}{1 + e^{(r-R)/a}}}$$

where R is the radius of nucleus a is a constant.

The quadrupole moment Q is defined as

$$\boxed{Q = \frac{1}{e}\int (3z^2 - r^2)\rho dV}$$

where the integration is carried out over the entire volume of the nucleus, e is the charge of proton, the nucleus is assumed to have a symmetry axis along z. The unit of Q is barn

1 barn = 10^{-28} m^2.

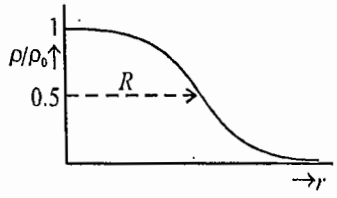

Figure 1.3: Variation of volume charge density of nucleus

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Special cases:

1. For spherically symmetric nucleus $Q = 0$.
2. For shape of prolate spheroid (the charge distribution streched in the Z direction) $Q > 0$ (positive).
3. For shape of oblate spheroid (the charge distribution is streched perpendicular), then the quadrupole moment $Q < 0$ (negative).

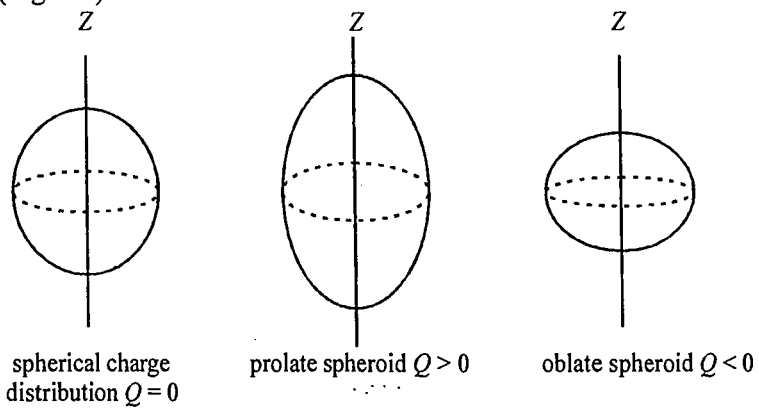

Figure 1.4: Quadrupole moment of a spherical ($b = a$), prolate ($b > a$) and oblate ($a > b$) spheroids

Spin of nucleus:

The spin of nucleus is the resultant spins of its constituent nucleons – protons and neutrons and have spin +½. The orienation of spin vector, according to wave mechanics is such that the components of spin can have its axis either parallel or antiparallel to a specific direction with spin angular momentum $+\frac{1}{2}\hbar$ or $-\frac{1}{2}\hbar$ respectively.

In addition to the spin angular momentum, the protons and neutrons in the nucleus have orbital angular momentum such that its magnitude in specific z-direction is an integral multiple $\hbar$. Thus, intrinsic angular momentum of a nucleus is a vector j such that

$$j = \sum_n \ell_n + \sum_s \ell_s = L + S$$

The magnitude of the total angular momentum vector j is given by

$$j = \sqrt{j(j+1)}\hbar$$

$j = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}\ldots\ldots$ for odd A type nuclei.

$j = 0, 1, 2, 3\ldots\ldots$ for even A type nuclei.

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Parity of nuclei:

Parity is the property of a wave function representing a quantum mechanical nuclear state, which may or may not change its sign on inversion of the space coordinates from (x, y, z) to $(-x, -y, -z)$ through origin.

$\psi(-x, -y, -z) = \psi(x, y, z)$ then parity is even.

$\psi(-x, -y, -z) = -\psi(x, y, z)$ then parity is odd.

Electric & magnetic moments of nucleus:

E_L and M_L are the electric and magnetic moments respectively, where L denotes the order, $L = 0$ for monopole, $L = 1$ for dipole, $L = 2$ for quadrupole and so on E_L has the parity $(-1)^L$ while M_L has parity $(-1)^{L+1}$. Following this for odd L only M_L exists, while for even L, only E_L exists. So, electric dipole, octapole are always zero. Similarly magnetic monopole, quadrupole are zero. We will see later that the only quantities of our interest are nuclear charge (electric monopole), magnetic dipole and electric quadrupole.

Nuclear forces:

We know that inside the nucleus there are proton and neutron. Protons repel each other by Coulombic repulsion and they should fly apart but it does not happen, this tells us that there is some kind of very strong attractive force. These forces are *N–N*, *P–P* and *N–P* force. These forces have following properties.

(i) These forces are short range forces
(ii) These forces are charge-independent
(iii) These forces are strongest known forces in nature
(iv) These forces are saturated by the surrounding nucleons
(v) These forces are spin-dependent
(vi) This force has non-central part (tensor nature of the strong force)
(vii) Velocity dependent.

SOLVED PROBLEMS

1. The stable nucleus that has $\frac{1}{3}$ the radius of ^{189}Os nucleus is, **[JEST 2015]**

(a) ^{7}Li (b) ^{16}O (c) $^{4}H_e$ (d) ^{14}N

Soln. Relation between radius of nucleus and atomic mass A is given by

$r \propto A^{1/3}$ ... (i)

Let, the radius of Os is r_{os} and radius of unknown nucleus is r_x.

Now given that, $r_x = \frac{1}{3} r_{os}$ ... (ii)

From equation (i) $r_{os} \propto (189)^{1/3} \Rightarrow r_{os} = k\,(189)^{1/3}$, ($k$ is proportionality constant) ... (iii)

And $r_x \propto (A_x)^{1/3}$ (Let A_x is the atomic mass of unknown nucleus) ... (iv)

$\Rightarrow r_x = k\,A_x^{1/3}$.

From equation (iii) and (iv), equation (ii)

$\Rightarrow k\,A_x^{1/3} = \frac{1}{3} k\,(189)^{1/3} \Rightarrow 3A_x^{1/3} = (189)^{1/3} \Rightarrow 27\,A_x = 189$ (taking cube on both side)

$\Rightarrow A_x = \frac{189}{27} = 7$

Therefore, atomic mass of stable nucleus is 7 i.e. the element is $^{7}_{3}Li$.

Correct option is (a)

2. The binding energy of the K-shell electron in a Uranium atom ($Z = 92$, $A = 238$) will be modified due to (i) screening caused by other electrons and (ii) the finite extent of the nucleus as follows:

(a) increases due to (i), remains unchanged due to (ii) **[JEST 2013]**
(b) decreases due to (i), decreases due to (ii)
(c) increases due to (i), increases due to (ii)
(d) decreases due to (i), remains unchanged due to (ii)

Soln. In case of screening caused by other electron, the binding energy of the k-shell electron will decrease. Also the finite extent of the nucleus will also decrease the binding energy.

Correct option is (b)

3. A proton is accelerated to a high energy E and shot at a nucleus of Oxygen $\left({}_{8}^{16}O\right)$. In order to penetrate the Coulomb barrier and reach the surface of the Oxygen nucleus, E must be at least **[TIFR 2012]**

(a) 3.6 MeV (b) 1.8 MeV (c) 45 keV (d) 180 eV

Soln. In order to penetrate the Coulomb barrier and reach the surface of the oxygen nucleus, E must be equal to the potential energy between $8e$ (nuclear charge of oxygen) and e (charge on proton),

$$\frac{(8e)e}{4\pi \in_0 r} = E \qquad \text{... (i)}$$

Now, $r \propto A^{1/3} \Rightarrow r = 1.2\times10^{-15}\times16^{1/3}$

Equation (i), $\Rightarrow \dfrac{8\times1.6\times10^{-19}\times1.6\times10^{-19}\times9\times10^{9}}{1.2\times10^{-15}\times16^{1/3}} = E$

$\Rightarrow \quad E = \dfrac{8\times2.56\times9\times10^{-29}}{1.2\times10^{-15}\times2\times\sqrt{2}} \Rightarrow E = 38.4\times1.414\times10^{-14}$ joule

$\Rightarrow \quad E = \dfrac{54.29\times10^{-14}}{1.6\times10^{-13}}$ MeV=3.4 MeV

Correct option is (a)

Note: The useful quantity to remember is $\dfrac{e^2}{4\pi\varepsilon_0} = \left(1.6\times10^{-19}\right)^2\times9\times10^{9}$ J-m

$$= \frac{\left(1.6\times10^{-19}\right)^2\times9\times10^{9}\ \text{eV-m}}{1.6\times10^{-19}} = 14.4\times10^{-10}\times10^{15}\ \text{eV-fm} = 1.44\ \text{MeV-fm}$$

4. A nucleus disintegrates at rest into two nuclear parts which have their velocities in the rest two nuclear parts which have their velocites in the ratio 3 : 2. The ratio of their nucleus sizes will be

(a) $3^{1/3}:2^{1/3}$ (b) $2^{1/3}:5^{1/3}$ (c) 2 : 3 (d) $2^{1/3}:3^{1/3}$

Soln. $\bigcirc \longrightarrow (1) + (2)$

$M \qquad m_1 \qquad m_2$

Velocity = 0 Velocity = V_1 Velocity = V_2

According to law of conservation of momentum

$\Rightarrow M(0) = m_1\vec{v}_1 + m_2\vec{v}_2$

$m_1v_1 = m_2v_2$

$\dfrac{m_2}{m_1} = \dfrac{v_1}{v_2}$

$\Rightarrow \quad \dfrac{m_2}{m_1} = \dfrac{3}{2} \qquad \text{... (1)}$

Mass of one atom $= \dfrac{\text{molecular mass } (M)}{\text{avogardro number } (A_0)}$

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From Eq. (1); $\frac{M_2/A_0}{M_1/A_0}=\frac{3}{2} \Rightarrow \frac{M_2}{M_1}=\frac{3}{2}$

And M α A { Here A is atomic mass }

So $\frac{A_2}{A_1}=\frac{3}{2}$

$$\left\{R=R_0A^{1/3} \Rightarrow \left(\frac{R}{R_0}\right)^3=A\right\} \Rightarrow \frac{\left(\frac{R_2}{R_0}\right)^3}{\left(\frac{R_1}{R_0}\right)^3}=\frac{3}{2} \Rightarrow \left(\frac{R_2}{R_1}\right)^3=\frac{3}{2} \Rightarrow \frac{R_1}{R_2}=\frac{2^{1/3}}{3^{1/3}}$$

Correct option is (d)

5. Calculate the electric potential energy due to the electric repulsion between two nuclei of ^{12}C when they 'touch' each other at surface

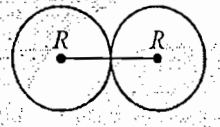
(a) 8.4 MeV (b) 4.6 MeV (c) 2.7 MeV (d) 10.2 MeV

Soln. Radius of a 12_C nucleus is

$R=R_0A^{1/3}$

$=(1.1fm)\ (12)^{1/3}$

$=2.52$ fm



The separartion between the centres of the nuclei is $2R=5.04$ fm

Here now the system is

$$U=\frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r};\ U=\frac{(9\times10^9)(6\times1.6\times10^{-19})^2}{5.04\times10^{-15}}$$

$U=1.64\times10^{-12}J$

$U=10.2MeV$

Correct option is (d)

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6. Find the maximum K.E energy that a beta particle can have in the following decay

$$^{176}\text{Lu} \rightarrow {}^{176}\text{Hf}+e+\bar{\nu}$$

Atomic mass of ^{176}Lu is 175.942694 u and that of ^{176}Hf is 175.941420 u

(a) 1.2 MeV (b) 0.8 MeV (c) 1 MeV (d) 1.4MeV

Soln. Here, Q = K.E (electron) + K.E (anti-neutrino)

So, $Q=\left[m\left(^{176}\text{Lu}\right)-m\left(^{176}\text{Hf}\right)\right]c^2$

So, $Q=(175.942694u-175.941420u)c^2=1.274\times10^{-3}uc^2$

So, { therefore, $1uc^2 \simeq 931MeV$ }

So, $Q=1.182$ MeV

Now, 1.182 MeV = K.E (electron) + K.E (anti-neutrino)

So, maximum K.E energy (electron) = 1.182

Assuming that anti-neutrono has zero K.E.

Correct option is (a)

7. The binding energy per nucleon of deutron $\left({}_1^2H\right)$ and Helium nucleus $\left({}_2^4He\right)$ is 1.1 MeV and 7 MeV respectively. If the two deutrons nuclei react to form a single nucleus, then the energy released is

(a) 12.4 MeV (b) 23.6 MeV (c) 28.2 MeV (d) 14.6 MeV

Soln. Hence, the nuclear reaction is ;

${}_1^2H + {}_1^2H \rightarrow {}_2^4He + x$

Acc. to law of conservation of energy

$$m({}_1^2H)c^2 + m({}_1^2H)c^2 = m({}_2^4He)c^2 + x$$

$$\Rightarrow \{m_pc^2 + m_nc^2 - B.E({}_1^2H) + m_pc^2 + m_nc^2 - B.E({}_1^2H)\}$$

$$= 2m_pc^2 + 2m_nc^2 - B.E\left({}_2^4Ne\right) + x$$

As $\{M\left({}_Z^AX\right)c^2 = Zm_pc^2 + (A-Z)m_nc^2 - B.E\}$

So, $-B.E\left({}_1^2H\right) - B.E\left({}_1^2H\right) = -B.E\left({}_2^4He\right) + x$

$$-1.1\times2 - 1.1\times2 = -7\times4 + x$$

$$x = 28.4 - 4.4$$

$$x = 23.6$$

Correct option is (b)

8. Two spherical nulci have mass numbers 125 and 64 with their radii 'R_1' and 'R_2' respectively. The ratio R_1/R_2 is

(a) 1.25 (b) 2.5 (c) 2 (d) 1.5

Soln. Therefore, $R = R_0A^{1/3}$

$$R_1 = R_0(125)^{1/3} \quad \ldots(1)$$

$$R_2 = R_0(64)^{1/3} \quad \ldots(2)$$

So $\frac{R_1}{R_2} = \frac{R_0(125)^{1/3}}{R_0(64)^{1/3}} \Rightarrow \frac{R_1}{R_2} = \frac{5}{4} \Rightarrow \frac{R_1}{R_2} = 1.25$

Correct option is (a)

9. The mean momentum $\overline{p}$ of a nucleon in a nucleus of mass number 'A' and atomic number 'Z' depends on A, Z as

(a) $\overline{p} \propto A^{1/3}$ (b) $\overline{p} \propto Z^{1/3}$ (c) $\overline{p} \propto A^{-1/3}$ (d) $\overline{p} \propto (AZ)^{-2/3}$

Soln. For a nucleon to be in Nucleus, maximum uncertainity in position = Diameter of nucleus.

So $(\Delta x)_{max} = D = 2R$.

Here R is radius of the nucleus and also from uncertainity prinicple, $\Delta x\Delta p = \hbar$

$\Delta p = \hbar/2R$ and also $R = R_0A^{1/3}$ So, $\Delta P = \left(\frac{\hbar}{2R_0}\right)A^{-1/3}$

Mean momentum of nucleon $\approx \Delta p$

Therefore, $\Delta p \propto A^{-1/3}$. So, mean momentum, $\overline{p} \propto A^{-1/3}$

Correct option is (c)

10.

Soln.

11.

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10. The graph of $\ln(R/R_0)$ versus $\ln A$. (R = radius of a nucleus and A = its mass a number) is

(a) Straight line (b) a parabola (c) an ellipse (d) None of these

Soln. $R = R_0 A^{1/3}$

$$\frac{R}{R_0} = A^{1/3} \quad \ldots (1)$$

Taking in both sides of Eq. (1),

$$\Rightarrow \ln\left(\frac{R}{R_0}\right) = \ln A^{1/3}$$

$$\Rightarrow \ln\left(\frac{R}{R_0}\right) = \frac{1}{3}\ln A \quad \ldots (2)$$

Eq. (2) represent a straight line, where y-axis is $\ln\left(R/R_0\right)$ and x-axis is $\ln A$ and slope is 1/3.

Correct option is (a)

11. A nucleus of rest mass M is initially in an excited state whose energy is ΔE above the ground state of the nucleus. The nucleus emits a gamma-ray of energy $h\nu$ and makes a transition to its ground state. Determine the fractional change $\frac{h\nu - \Delta E}{\Delta E}$. [You may assume $\Delta E < Mc^2$]

(a) $\frac{\Delta E}{2Mc^2}$ (b) $\frac{\Delta E^2}{Mc^2}$ (c) $\frac{\Delta E^2}{M^2c^4}$ (d) $-\frac{\Delta E}{2Mc^2}$

Soln. The nucleus will recoil when it emits a γ-ray because of the conservation of momentum. It will thereby acquire some recoil energy from the excitation energy and make $h\nu$ less than ΔE.

Let the total energy of the nucleus be E and its recoil momentum be p. The conservation of energy and of momentum give $\Rightarrow$ momentum of photon = momentum of nucleus.

Initial energy, $E_i = Mc^2 + \Delta E$

Final energy, $E_f = p_\gamma c + \sqrt{M^2c^4 + p_\gamma^2 c^2}$

$$E_f = E_\gamma + \sqrt{M^2c^4 + E_\gamma^2}$$

From law of conservation of energy $E_\gamma + \sqrt{M^2c^4 + E_\gamma^2} = Mc^2 + \Delta E$

$$\Rightarrow \quad \sqrt{M^2c^4 + E_\gamma^2} = \left(Mc^2 + \Delta E\right) - E_\gamma$$

$$\Rightarrow \quad M^2c^4 + E_\gamma^2 = \left(Mc^2 + \Delta E\right)^2 + E_\gamma^2 - 2E_\gamma\left(Mc^2 + \Delta E\right)$$

$$\Rightarrow \quad 2E_\gamma\left(Mc^2 + \Delta E\right) = \left(Mc^2 + \Delta E\right)^2 - M^2c^4$$

$$\Rightarrow \quad 2E_\gamma\left(Mc^2 + \Delta E\right) = M^2c^4 + \Delta E^2 + 2Mc^2\Delta E - M^2c^4$$

$$E_\gamma = \frac{(\Delta E)^2 + 2Mc^2\Delta E}{2(Mc^2 + \Delta E)}$$

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We have

$$E_\gamma = \frac{1}{2Mc^2}\cdot\frac{(\Delta E)^2 + 2Mc^2\Delta E}{\left(1+\frac{\Delta E}{Mc^2}\right)} \approx \Delta E - \frac{(\Delta E)^2}{2Mc^2},$$

or $$\frac{h\nu - \Delta E}{\Delta E} = -\frac{\Delta E}{2Mc^2}.$$

Correct option is (d)

12. Given that the binding energy per nucleon of an α-particle is 7 MeV and that the energy release in the reaction $d + d \to \alpha$ is 23.6 MeV, total binding energy of a deutron is

(a) 1.1 MeV (b) 2.2 MeV (c) 3.3 MeV (d) 4.4 MeV

Soln. $M\left({}^{A}_{Z}X\right)c^2 = Zm_pc^2 + (A-Z)m_nc^2 - \text{B.E.}$

So, $d + d \to \alpha + 23.6$ MeV ...(1)

From law of conservation of energy

$\Rightarrow$ $M(d)c^2 + M(d)c^2 = M(\alpha)c^2 + 23.6$

$\Rightarrow$ $m_pc^2 + m_nc^2 - \text{B.E}(d) + m_pc^2 + m_nc^2 - \text{B.E}(d) = 2m_pc^2 + 2m_nc^2 - 7\times 4 + 23.6$

$\Rightarrow$ $-2\times\text{B.E}(d) = -28 + 23.6$

$-2\times\text{B.E}(d) = -4.4$

$\text{B.E}(d) = 2.2$

Correct option is (b)

13. The binding energies of ^{1}H, ^{4}He and ^{7}Li are B_1, B_4 and B_7 respectively. The Q-value of the reaction :

${}^1\text{H} + {}^7\text{Li} \longrightarrow 2\,{}^4\text{He} + Q$ is given by

(a) $2B_1 + 4B_7 - 6B_4$ (b) $2B_4 - B_1 - B_7$ (c) $B_4 - B_1 - B_7$ (d) $-B_1 + 7B_1 - 8B_4$

Soln. $B.E.\left({}^{A}_{Z}X\right) = Z\,m_pc^2 + (A-Z)m_nc^2 - m\left({}^{A}_{Z}X\right)c^2$;

So, according to law of conservation of energy

$$m\left({}^1\text{H}\right)c^2 + m\left({}^7\text{Li}\right)c^2 = 2m\left({}^4\text{He}\right)c^2 + Q$$

$\Rightarrow$ $m_pc^2 - B_1 + 3m_pc^2 + 4m_nc^2 - B_7 = 2\left\{2m_pc^2 + 2m_nc^2 - B_4\right\} + Q$

$\Rightarrow$ $4m_pc^2 + 4m_nc^2 - B_1 - B_7 = 4m_pc^2 + 4m_nc^2 - 2B_4 + Q$

$\Rightarrow$ $Q = 2B_4 - B_1 - B_7$

Correct option is (b)

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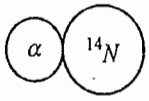
14. Rutherford bombarded 7.7 MeV α-particles on $^{14}N_7$ to initiate the nuclear reactions $^{14}N(\alpha, p)^{17}O$. The height of coulomb barrier faced by α-particle is about (Given : $r_0 = 1.2$ fm)

(a) 10.6 MeV (b) 2.1 MeV (c) 4.2 MeV (d) 8.4 MeV

Soln. The height of coulomb barrier depends on the electrostatic repulsion coulomb energy.



Here radius of α-particle, $R_1 = R_0(4)^{1/3}$ and radius of ^{14}N particle, $R_2 = R_0(14)^{1/3}$.

So, net distance between α-particle and ^{14}N is $R = R_1 + R_2$

$$R = R_0(14^{1/3} + 4^{1/3}) \quad \ldots(1)$$

Therefore, $E_{\text{electrostatic}} = \dfrac{1}{4\pi\varepsilon_0} \times \dfrac{(2e)(7e)}{R_0(14^{1/3} + 4^{1/3})}$

$$E_{\text{electrostatic}} = \frac{9\times10^9 \times 2\times1.6\times10^{-19}}{1.2\times10^{-15}\times(14^{1/3}+4^{1/3})\times10^6}\ \text{MeV}$$

$\Rightarrow$ $E_{\text{electrostatic}} = 4.2$ MeV.

Correct option is (c)

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PRACTICE SET

1. The experimentally measured nuclear charge density distribution ρ is approximated by the function:

(a) $\rho = \rho_0\left(1+e^{(r-R)/a}\right)$ (b) $\rho = \frac{\rho_0}{1+e^{(r-R)/a}}$ (c) $\rho = \rho_0 e^{-r/R}$ (d) $\rho = \frac{\rho_0}{r}e^{-r/R}$

2. The radius of a $^{64}_{29}Cu$ nucleus is measured to be 4.8×10^{-13} cm. The root-mean-square (rms) energy of a nucleon in a nucleus of atomic number A in its ground state varies as:

(a) $A^{4/3}$ (b) $A^{1/3}$ (c) $A^{-1/3}$ (d) $A^{-2/3}$

3. The ratio of radii of the nuclei He^4 and S^{32} is :

(a) $\frac{2}{1}$ (b) $\frac{3}{2}$ (c) $\frac{1}{2}$ (d) $\frac{4}{1}$

4. The binding energy per nucleon is approximately constant this leads to :
(a) saturation property of nuclear forces (b) short range nature
(c) non-central nature (d) spin dependency.

5. Number of nucleon/m^3 in a nucleus is :
(a) 10^{40} (b) 10^{44} (c) 10^{50} (d) 10^{49}

6. Binding energy per nucleon is max. for
(a) He^4 (b) Ca^{40} (c) Fe^{56} (d) Pb^{206}

7. The nuclear density in kg/mm^3 is:
(a) 10^{17} (b) 10^{13} (c) 10^{8} (d) 10^{10}

8. The deviation of the charge distribution of a nucleus from spherical symmetry can be estimated by measuring its:
(a) electric charge (b) electric dipole
(c) magnetic dipole moment (d) electric quadrupole moment

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ANSWER KEY					
Questions	1	2	3	4	5
Option	(b)	(c)	(c)	(a)	(b)
Questions	6	7	8		
Option	(c)	(c)	(d)		

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Chapter 2

Nuclear Models

Introduction:

In previous chapter, we have studied about nuclear properties like variation of radius with mass number, charge density, binding energy, etc. But to understand the above behaviour of atomic nucleus, it is essential to know the exact nature of the forces between the nucleons. However, due to lack of knowledge of the exact internucleon force, it has not been possible to develop a satisfactory theory of nuclear structure. In absence of a comprehensive theory, attempts were made to develop nuclear models to explain the properties of nuclei. Several different models have been suggested, each with its successes and limitations. The important models are

(a) The liquid drop model
(b) Nuclear shell model
(c) Fermi gas model
(d) Alpha model
(d) Collective model
(e) Optical model.

A. Liquid drop model:

The liquid drop model of the nucleus was first proposed by Niels Bohr and F. Kalcar in the year 1937. They observed that there exists many similarities between the drop of a liquid and a nucleus. For instance,

(i) both the liquid drop and the nucleus possess constant density,
(ii) The constant binding energy per nucleon of a nucleus is similar to the latent heat of vaporization of a liquid,
(iii) The evaporation of a drop corresponds to the radioactive properties of the nucleus, and
(iv) The condensation of drops bears resemblance with the formation of compound nucleus, etc.

- According to this model, the nucleus is supposed to be spherical in shape in the stable state with radius $R = R_0 A^{1/3}$, just as a liquid drop is spherical due to symmetrical surface tension forces. The surface tension effects are analogous to the potential barrier effects on the surface of the nucleus.
- The density of a liquid drop is independent of the volume, as is the case with the nucleus. But whereas the nuclear density is independent of the type of nucleus, the density of a liquid does depend on its nature.
- Like the nucleons inside the nucleus, the molecules in the liquid drop interact only with their immediate neighbours.
- The non-independence of the binding energy per nucleon on the number of nucleons in the nucleus is analogous to the non-independence of the heat of vaporization of a liquid drop on the size of the drop.
- Molecules in a liquid drop evaporate from the liquid surface in raising the temperature of the liquid due to their increased energy of thermal agitation. Analogously, if high energy nuclear projectiles (or photons) bombard the nucleus, a compound nucleus is formed in which the nucleons quickly share the incident energy and the emission of nucleons (or photon) occurs.

- The phenomenon of nuclear fission is easily explained as the splitting of the liquid drop if set into vibration with sufficient energy.

Semi-empirical binding energy or mass formula:

C.V. Weizsacker, a German physicist, proposed the following semi-empirical formula for the nuclear binding energy B.E. (in MeV) for the nucleus (Z, A)

$$B.E. = a_v A - a_s A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}} - a_n \frac{(A-2Z)^2}{A} (\pm, 0) \frac{\delta}{A^{3/4}}$$

with the constants having the value,

$a_v = 15.75, a_s = 17.80, a_c = 0.71, a_n = 22.7$ and $\delta = 33.6$ all are in MeV.

These values are not absolute other sets of values are also possible.

(i) Volume energy:

The first term, $E_v = a_v A$, is the volume effect representing the volume energy of all nucleous. The more the total number of nucleons A, more difficult it becomes to remove an individual nucleon from the nucleus. Since the nuclear density is nearly constant, the nuclear mass is proportional to the nuclear volume, which is again proportional to R^3. But $R \propto A^{1/3} \Rightarrow R^3 \propto A$. So, the volume energy $E_v \propto A$.

$\Rightarrow \quad E_v = a_v A$

This energy corresponds to the amount of heat energy (the heat of vaporisation) required to transfonn a liquid to its vapour state being proportional to the mass of the liquid.

(ii) Surface energy:

The second term, $E_s = a_s A^{2/3}$, is the surface effect being similar to the surface tension in liquids, like the molecules on the surface of a liquid, the nucleons at the surface of the nucleus are not completely surrounded by other nucleons. It results in reducing the total binding energy due to nucleons on the surface. This correction due to surface energy E_s, which is proportional to the surface area of the nucleus i.e. to $4\pi R^2$ i.e. $E_s \propto R^2 \quad \Rightarrow \quad E_s \propto A^{2/3} \quad \Rightarrow \quad E_s = a_s A^{2/3}$

(iii) Coulomb energy:

The third term, E_c, is the Coulomb electrostatic repulsion between the charged particles in the nucleus. Since each charged particle repulses all the other charged particles, this term would be directly proportional to the possible number of combinations for a given proton number Z, which is $Z(Z-1)/2$. The energy of interaction between protons is again inversely proportional to the distance of separation R, so the energy associated with Coulomb repulsion is:

$$E_c = k \frac{Z(Z-1)}{R} = a_c \frac{Z(Z-1)}{A^{1/3}}$$

where, $a_c = \frac{3}{5} \times \frac{e^2}{4\pi\varepsilon_0 R_0}$; binding energy decreases due to Coulomb terms.

(iv) Asymmetry energy:

The fourth term E_a originates from the asymmetry between the number of protons and the number of neutrons in the nucleus. Nuclear data for stable nuclei indicate that for lighter nuclei, the number of protons is almost equal to that of neutrons: $N = Z$. As A increases, the symmetry of proton and neutron number is lost and the number of neutrons exceeds that of protons to maintain the nuclear stability. This excess of neutrons over protons, i.e. N–Z, is the measure of the asymmetry and it decreases the stability or the B.E. of medium or heavy nuclei.

So, $\quad E_a \propto (N-Z)$, and $E_a \propto (N-Z)/A \quad \Rightarrow \quad E_a = a_n \frac{(N-Z)^2}{A} = a_n \frac{(A-2Z)^2}{A}$

(v) Pairing energy:

The last term, a pure corrective term, is called pairing energy E_p.

$$E_p = (\pm, 0)\frac{\delta}{A^{3/4}}$$

Z	N	A	δ	E_p
even	even	even	34	$+\delta / A^{3/4}$
even	odd	odd	0	0
odd	even	odd	0	0
odd	odd	even	35	$-\delta / A^{3/4}$

- Binding fraction (Or binding energy per nucleon),

$$f_B = \frac{B.E.}{A} = a_v - \frac{a_s}{A^{1/3}} - \frac{a_c Z(Z-1)}{A^{4/3}} - a_n \frac{(A-2Z)^2}{A^2}(\pm, 0)\frac{\delta}{A^{7/4}}$$

Mass parabola and stability of nuclei against β-decay:

- Mass of nucleus, $M(A,Z) = Z M_p + (A-Z) M_n - B.E./c^2$

$$= ZM_p + (A-Z)M_n - \frac{1}{c^2}\left[a_V A - a_s A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}} - a_n \frac{(A-2Z)^2}{A}(\pm, 0)\frac{\delta}{A^{3/4}}\right]$$

The above formula is known as the semi-empirical mass formula.

Assuming, $F_A = A(M_n - a_v + a_n) + a_s A^{2/3}$

$$p = -4a_n - (M_n - M_p);\ q = \frac{1}{A}\left(a_c A^{2/3} + 4a_n\right)$$

$\Rightarrow$ $\boxed{M(A,Z) = F_A + pZ + qZ^2}$, it is equation of parabola for given A is called mass parabola.

$\Rightarrow$ $\left(\frac{\partial M}{\partial Z}\right)_A = p + 2qZ = 0$ at $Z = Z_A$

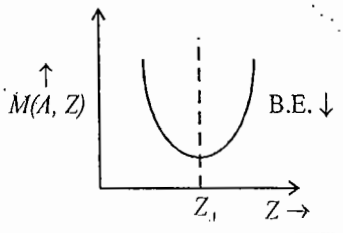
$\Rightarrow$ $Z_A = -\frac{p}{2q} = \frac{(M_n - M_p + 4a_n)A}{2\left(a_c A^{2/3} + 4a_n\right)}$ $\Rightarrow$ $Z_A = A/\left(1.98 + 0.015A^{2/3}\right)$

In most cases, the value of Z nearest to Z_A gives the actual stablest nucleus for a given A.

All isobars having mass greater than that of stable isobar. They will decay by emission of β^-, β^+ and k capture.

1. The isobar to the left of the stable one decay by β^- emission.

2. The isobar to the right of the stable one decay by β^+ emission or electron-capture or by both.



Magic numbers : If the number of proton or neutron or both are 2, 8, 20, 28, 50, 82, 126 of a nucleous then it is very stable. These numbers are called as Magic numbers.

B. Shell model:

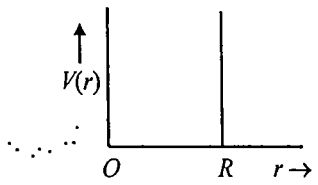
Liquid drop model can not explore why some nucleus having proton or neturon or both are 2, 8, 20, 28, 50, 82, 126 they have high binding energy than others. The scientists thought that there was some arrangement like electron distribution in the atom.

Since we do not know the exact force nature inside the nucleus so we need to take a guess.

In shell model we can take any of the following three potentials.

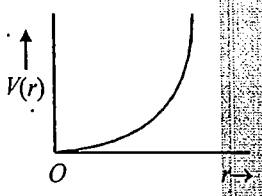
1. Three dimensional infinite spherical well potential

$$V(r)=\begin{cases}0 & \text{for } r \le R\\ \infty, & \text{for } r > R\end{cases}$$



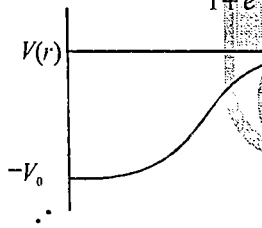
2. Three dimensional harmonic oscillator potential

$$V(r)=\frac{1}{2}kr^2, \text{ where } k>0$$



3. Saxon-Woods model potential

$$V(r)=-\frac{V_0}{1+e^{(r-R)/a}}$$



where, $V_0 \simeq 50$ MeV, $R=r_0A^{1/3}$, $a=0.52$ fm

In all the above three types of potentials, we get the energy levels E_{nl}. For Saxon-Woods potential energy E_{nl} is given as

$$E_{n\ell}=\frac{\hbar^2\ell(\ell+1)}{2\mu R_0^2}\left(1+\frac{12a^2}{R_0^2}\right)-\frac{\hbar^2}{2\mu a^2}\left\{\frac{\left[\sqrt{1+192\ell(\ell+1)a^4}-2n-1\right]^2}{16}+\frac{4\left[\frac{\mu a^2V_0}{\hbar^2}-\frac{4\ell(\ell+1)a^3}{R_0^3}\right]^2}{\left[\frac{\sqrt{1+192\ell(\ell+1)a^4}}{R_0^4}-2n-1\right]^2}+\frac{\mu V_0a^2}{\hbar^2}\right\}$$

The lowest level is $1s$(i.e. $n=1$, $l=0$) which can contain up to 2 protons or neutrons. Then comes $1p$ which can contain up to a further 6 protons. This explains the first two magic numbers (2 and 8). Then there is a level $1d$ but this is quite close in energy to $2s$ so that they form the same shell. This allows a further 2 + 10 protons or neutrons giving us the next magic number of 20.

The next two levels are $1f$ and $2p$ which are also quite close together and allow a further 6 + 14 protons. This would suggest that the next magic number was 40 but experimentally it is known to be 50.

Spin-orbit potential:

To explain magic numbers, spin-orbit correction term $f(r)\vec{L}\cdot\vec{S}$, where $f(r)$ is an fuction of r, $f(r)$ may be taken as a positive constant.

Therefore, the potential become

$$V_{tot} = V(r) - f(r)\vec{L}\cdot\vec{S}$$

As in the case of atomic physics (L-S coupling scheme) the orbital and spin angular momentum of the nucleous combine to give a total angular momentum J which can take the values $j = \ell + \frac{1}{2}$ or $j = \ell - \frac{1}{2}$.

For $j = \ell + \frac{1}{2}, \langle J^2\rangle = \langle L^2\rangle + \langle S^2\rangle + 2\langle \vec{L}\cdot\vec{S}\rangle \Rightarrow \langle \vec{L}\cdot\vec{S}\rangle = \frac{1}{2}\left[\langle J^2\rangle - \langle L^2\rangle - \langle S^2\rangle\right]$

Where, $\langle J^2\rangle = j(j+1)\hbar^2$,

$$\langle L^2\rangle = \ell(\ell+1)\hbar^2;\ \langle S^2\rangle = s(s+1)\hbar^2 = \frac{1}{2}\left(\frac{1}{2}+1\right)\hbar^2 = \frac{3}{4}\hbar^2$$

For $J = \ell + \frac{1}{2}$

$$\langle \vec{L}\cdot\vec{S}\rangle = \frac{\hbar^2}{2}\left[\left(\ell+\frac{1}{2}\right)\left(\ell+\frac{3}{2}\right) - \ell(\ell+1) - \frac{3}{4}\right]$$

$$= \frac{\hbar^2}{2}\left[\ell^2 + \frac{3}{2}\ell + \frac{1}{2}\ell + \frac{3}{4} - \ell^2 - \ell - \frac{3}{4}\right] = \frac{\hbar^2}{2}[\ell] = \frac{\hbar^2\ell}{2}$$

For $J = \ell - \frac{1}{2}$

$$\langle \vec{L}\cdot\vec{S}\rangle = \frac{\hbar^2}{2}\left[j(j+1) - \ell(\ell+1) - s(s+1)\right]$$

$$= \frac{\hbar^2}{2}\left[\left(\ell-\frac{1}{2}\right)\left(\ell+\frac{1}{2}\right) - (\ell+1) - \frac{3}{4}\right] = \frac{\hbar^2}{2}\left[\ell^2 - \frac{1}{4} - \ell^2 - \ell - \frac{3}{4}\right] = -\frac{\hbar^2}{2}(\ell+1)$$

We see that, $j = \ell + \frac{1}{2}$ have lower energy and $J = \ell - \frac{1}{2}$ have higher energy and energy splitting

$$\Delta E = \frac{\hbar^2}{2}(\ell+1) - \left(-\frac{\hbar^2\ell}{2}\right)$$

$$\Delta E = \frac{\hbar^2}{2}(2\ell+1) = \left(\ell+\frac{1}{2}\right)\hbar^2 \quad \text{i.e. higher } l \text{ more energy gap.}$$

We see that this larger spin-orbit effect leads to crossing over of energy levels into different shells. For example the state above the $2p$ state is $1g(l=4)$, which splits into $1g_{9/2}\left(J = \ell + \frac{1}{2}\right)$ and $1g_{7/2}\left(J = \ell - \frac{1}{2}\right)$.

The energy of the $1g_{9/2}$ state is sufficiently low that it joins the shell below. So that this forth shell now consists of $1f_{7/2}, 2p_{3/2}, 1f_{5/2}, 2p_{1/2}$ and $1g_{9/2}$. The maximum occupancy of j state ($(2J+1)$ proton or

neutron for each j) is now $8 + 4 + 6 + 2 + 10 = 30$ which added to the previous magic number 20, gives the next observed magic number of 50.

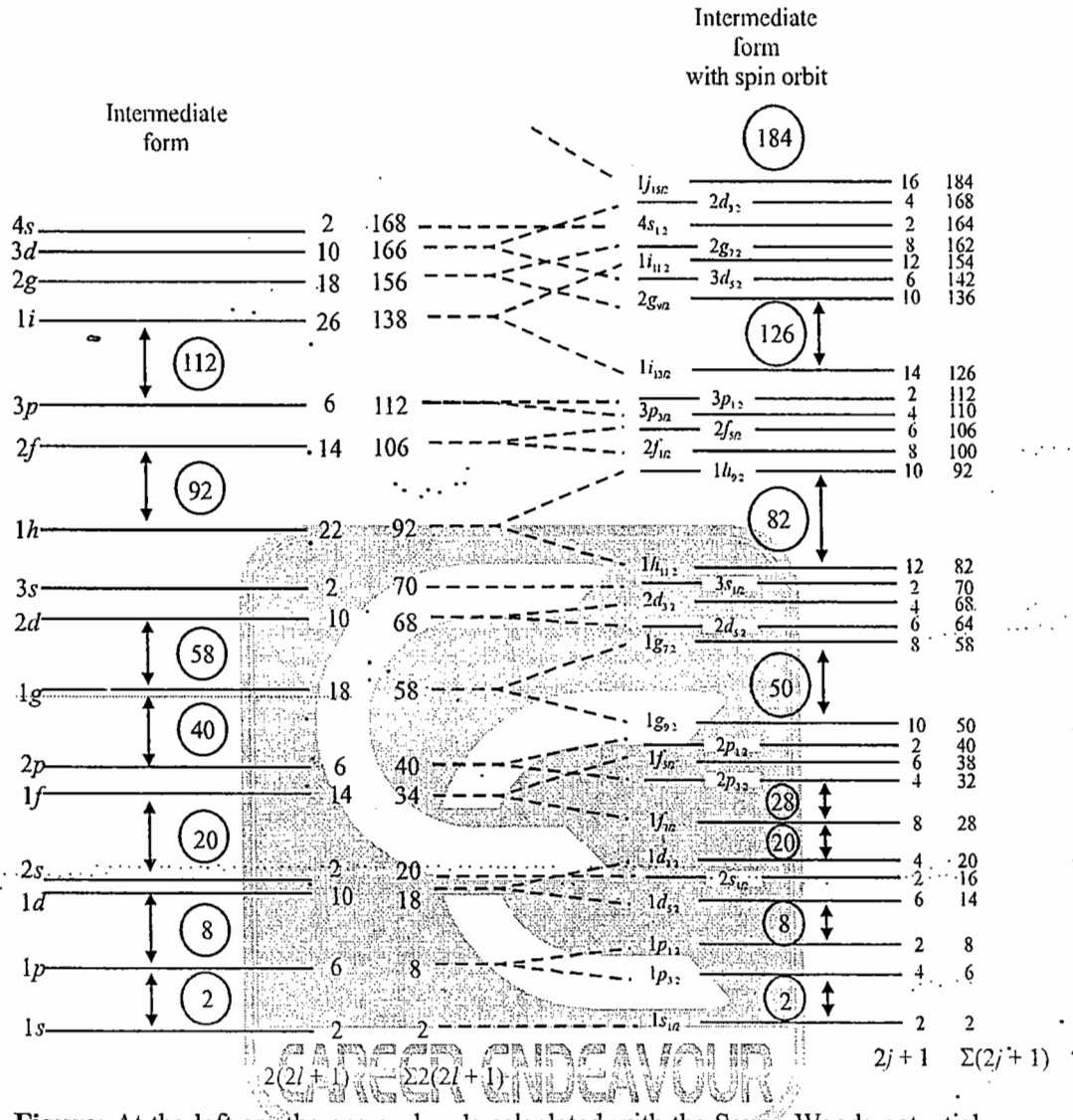


Figure: At the left are the energy levels calculated with the Saxon-Woods potential. To the right of each level are shown its capacity and the cumulative number of nucleons upto that level. The right side of the figure shows the effect of the spin-orbit interaction, which splits the levels with $t > 0$ into two new levels. The shell effect is quite apparent, and the magic numbers are exactly reproduced.

Success of shell model:

1. It very well explains the existence of magic numbers and the stability and high binding energy on the basis of closed shells.
2. The shell model provides explanation for the ground state spins and magnetic dipole moment of the nuclei. The neutrons and protons with opposite spins pair off so that the mechanical and magnetic dipole moment cancel and the odd or left out proton or neutron contributes to the spin and magnetic moment of the nuclei as a whole.
3. Nuclear isomerism, i.e., existence of isobaric, isotopic nuclei in different energy states of odd-A nuclei between 39-49, 69-81, 111 to 125 has been explained by shell model by the large difference in nuclear spins of isomeric states are their A-values are close to magic numbers.

The nuclear shell structures of protons and neutrons are shown in the following figure.

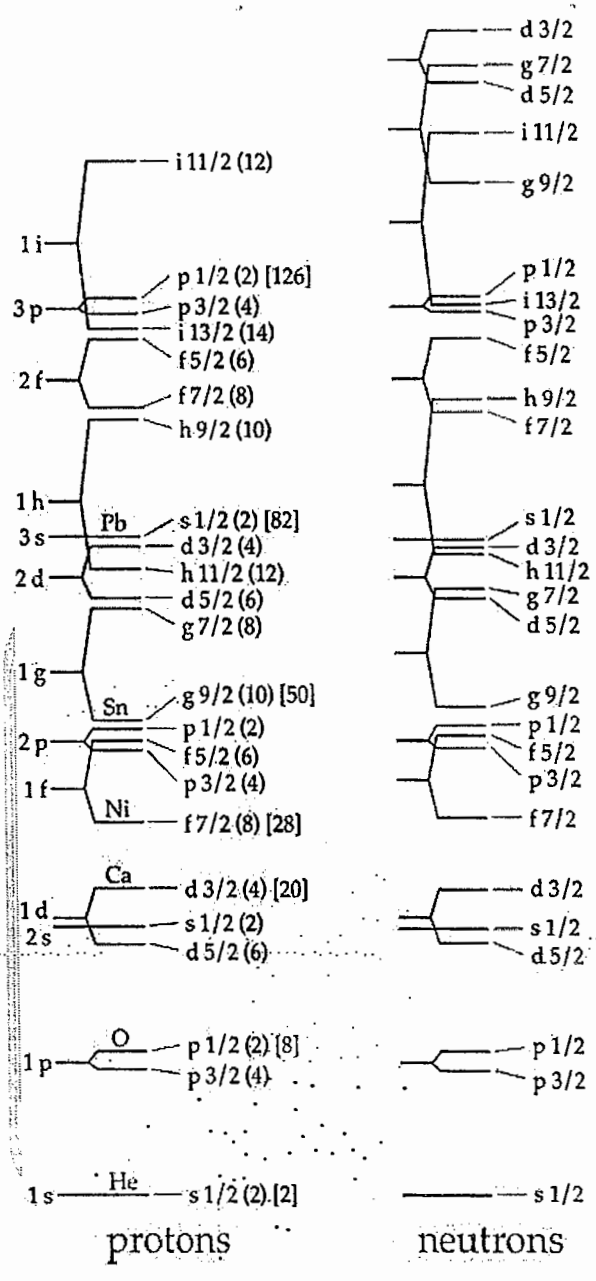


Fig.: Nuclear shell structure for protons and neutrons

Applications of Nuclear Shell Model:

(1) Determination of total angular momentum and parity of the ground state

1. If protons and neutrons both are even, then the total angular momentum become zero and parity is even.
2. If one of the nucleon (proton or neutron) is even and other is odd then the j value and $(-1)^{\ell}$ of the last unpaired nucleon give the total angular momentum and parity.
3. For odd-odd nucleon we use Brennan-Bernstein rules. According to this rules we define the quantity, $N = (j_P - \ell_P) + (j_N - \ell_N)$

   (i) If $N = 0$, then $j = |j_P - j_N|$, parity $= -ve$

   (ii) If $N = \pm 1$ and $N = Z$ then $J = |J_P + J_N|$, parity $= \pm 1$

   (iii) If $N = \pm 1$ and $N \neq Z$ then $J = (J_P + J_N - 1)$, parity $= +ve$

Example : $_8O^{16}$

$$Z = 8 \text{ (even)} \rightarrow 1s^2_{1/2}\, 1p^4_{3/2}\, 1p^2_{1/2}$$

$$N = 8 \text{ (even)} \rightarrow 1s^2_{1/2}\, 1p^4_{3/2}\, 1p^2_{1/2}$$

Since, there have no unpaired nucleon.

Therefore, parity is positive and angular momentum $j = 0$. It is represented by 0^+.

$^{27}_{13}Al$: $Z = 13(\text{odd}) \rightarrow 1s^2_{1/2}\, 1p^4_{3/2}\, 1p^2_{1/2}\, 1d^5_{5/2}$

$N = 14(\text{even}) \rightarrow$ does not contribute in angular momentum

Therefor, $j = 5/2$; $l = 2$ (for d, l = z)

Parity = $(-1)^2 = +1$, even parity. It is represented by $\frac{5^+}{2}$.

$^{33}_{16}S$: $Z = 16(\text{even}) \rightarrow$ does not contribute in angular momentum

$N = 17(\text{odd}) \rightarrow 1s^2_{1/2}\, 1p^4_{3/2}\, 1p^2_{1/2}\, 1d^5_{5/2}\, 2s^2_{1/2}\, 1d^2_{3/2}$

Therefor, $j = 3/2$; $l = 2$ (for d, $l = 2$)

Parity = $(-1)^2 = +1$, even parity. It is represented by $\frac{3^+}{2}$.

$^{38}Cl_{17}$:

$Z = 17(\text{odd}) \rightarrow 1s^2_{1/2}\, 1p^4_{3/2}\, 1p^2_{1/2}\, 1d^6_{5/2}\, 2s^2_{1/2}\, 1d^1_{3/2}$ (particle state)

$N = 21(\text{odd}) \rightarrow 1s^2_{1/2}\, 1p^4_{3/2}\, 1p^2_{1/2}\, 1d^6_{5/2}\, 2s^2_{1/2}\, 1d^4_{3/2}\, 1f^1_{7/2}$ (particle state)

$$N = (j_P - \ell_P) + (j_N - \ell_N) = -\frac{1}{2} + \frac{1}{2} = 0$$

$$\therefore \quad j = |j_P - j_N| = \left|\frac{3}{2} - \frac{7}{2}\right| = \frac{4}{2} = 2 \text{ and parity} = -1$$

Therefore, spin parity = 2^-.

$^{26}Al_{13}$:

$Z = 13(odd) = 1s^2_{1/2}\, 1p^4_{3/2}\, 1p^2_{1/2}\, 1d^5_{5/2}$

$N = 13(odd) = 1s^2_{1/2}\, 1p^4_{3/2}\, 1p^2_{1/2}\, 1d^5_{5/2}$

$$N = (J_P - \ell_P) + (J_N - \ell_N) = \frac{1}{2} + \frac{1}{2} = 1 \quad \text{and } N = Z$$

$$J = |J_P + J_N| = \left|\frac{5}{2} + \frac{5}{2}\right| = 5, \text{ parity} = +\text{ve} \text{ and parity is +ve}$$

Therefore, spin parity 5^+.

$^{56}Co_{27}$:

$Z = 27(odd) = 1s^2_{1/2}\, 1p^4_{3/2}\, 1p^2_{1/2}\, 1d^6_{5/2}\, 2s^2_{1/2}\, 1d^4_{3/2}\, 1f^7_{7/2}$

(hole state, because only one particle state is empty)

$N = 29(odd) = 1s^2_{1/2}\, 1p^4_{3/2}\, 1p^2_{1/2}\, 1d^6_{5/2}\, 2s^2_{1/2}\, 1d^4_{3/2}\, 1f^8_{7/2}\, 2p^1_{3/2}$ (particle state)

CAREER ENDEAVOUR

$$N = (j_P - \ell_P) + (j_N - \ell_N) = \frac{1}{2} + \frac{1}{2} = 1 \text{ and } N \neq Z$$

$$\therefore \quad J = |j_P + j_N - 1| = \frac{7}{2} + \frac{3}{2} - 1 = 4, \text{ parity} = +ve$$

Therefore, spin parity = 4^+.

(2) Determination of Magnetic Moments of Nuclei:

The magnetic moment of a nucleus is the vector sum of the spin magnetic moment ($\vec{\mu}_s$) and orbital magnetic moment ($\vec{\mu}_l$). Thus,

$$\vec{\mu} = \vec{\mu}_s + \vec{\mu}_l \qquad \ldots (i)$$

where, $\vec{\mu}_s$ is the vector sum of the intrinsic magnetic moments of the individual nucleons in the nucleus. The intrinsic moments for protons and neutrons are

$$\mu_p = g_p \frac{\mu_N}{2} \text{ and } \mu_n = g_n \frac{\mu_N}{2} \qquad \ldots (ii)$$

where, $\mu_N = \frac{e\hbar}{2m_p}$ is the nuclear magneton and m_p being the mass of proton. g_n and g_p are the gyromagnetic ratios for the neutron and proton respectively and their numerical values are

$$g_n = -2 \times 1.9131 \text{ and } g_p = 2 \times 2.7927 \qquad \ldots (iii)$$

One can calculate the magnetic moment of an odd nucleus with the help of extreme single particle shell model. We can write the magnetic moment of a nucleus of spin I, i.e. total angular momentum as

$$\mu_I = g_I \sqrt{I(I+1)}\, \mu_N \qquad \ldots (iv)$$

A magnetic field has to be applied while measuring the magnetic moments and it is the component of μ_I in the field direction Z which is determined. Applying space quantization rule, equation (iv) takes the form

$$\mu_Z = \mu_I \cos(I, B) = \frac{\mu_I m_I}{\sqrt{I(I+1)}}, \text{ where } (I, B) \text{ is angle between } I \text{ and } B \qquad \ldots (v)$$

where, μ_I is the magnetic quantum number and can take up the values $\mu_I = I, (I-1), \ldots, -I$, and B is the magnetic induction field. The component corresponding to $\mu_I = I$ is the largest and gives the measured magnetic moment

$$\mu_Z = \frac{\mu_I I}{\sqrt{I(I+1)}} = g_I I\, \mu_N \qquad \ldots (vi)$$

From the above, it is clear that in the extreme single particle shell model, any one kind always gives the resultant spin ($I = 0$). Obvisouly, the magnetic moment of an even-even nucleus will be zero, i.e., $(\mu_I)_{ee} = 0$

This means that in an odd A nucleus, it is the last odd nucleon (proton or neutron) determines the magnetic moment. Obviously, for such a nucleus, $I = j$, where j is the total angular momentum of last unpaired nucleon.

In order to get the magnetic moment, we have to add both the intrinsic magnetic moment ($\vec{\mu}_s$) and the magnetic moment due to its orbital motion ($\vec{\mu}_l$) i.e., $\vec{\mu}_j = \vec{\mu}_l + \vec{\mu}_s$, where $\mu_s = \mu_p$ for the proton and $\mu_s = \mu_n$ for the neutron.

The orbital motion of a nucleon having azimuthal angular momentum produces a magnetic moment,

$$\mu_l = g_e \mu_N \sqrt{l(l+1)} \quad \text{... (vii)}$$

Now, since the neutron is an uncharged particle, its orbital motion does not produce any magnetic moment, i.e., $g_l = 0$. Thus,

$$(\mu_l)_n = 0 \quad \text{... (viii)}$$

One can write $g_l = 1$ in the case of proton. Thus, the orbital contribution is

$$(\mu_{lp}) = \mu_N \sqrt{l(l+1)} \quad \text{... (ix)}$$

Nucleons are spin $\frac{1}{2}$ particles and hence one can write the quantum mechanical values of intrinsic magnetic moment as

$$\mu_s = g_s \mu_N \sqrt{s(s+1)} \quad \text{... (x)}$$

where, $s = \frac{1}{2}$ is the spin quantum number, $g_s = g_p$ for the proton and $g_s = g_n$ for the neutron. In the direction of J, the total magnetic moment is

$$m_l = m_j = \mu_l \cos(l, j) + \mu_s \cos(s, j)$$

$$= \mu_N \left[g_l \sqrt{l(l+1)} \cos(l, j) + g_s \sqrt{s(s+1)} \cos(s, j) \right] \quad \text{... (xi)}$$

Using equation (iv), one can write

$$\mu_j = g_j \sqrt{j(j+1)}\, \mu_N \quad \text{... (xii)}$$

From the cosine rule, we have,

$$\cos(l, j) = \frac{j(j+1) + l(l+1) - s(s+1)}{2\sqrt{j(j+1)} \cdot \sqrt{l(l+1)}} \text{ and } \cos(s, j) = \frac{j(j+1) + s(s+1) - l(l+1)}{2\sqrt{j(j+1)} \cdot \sqrt{s(s+1)}}$$

Thus, we have

$$\mu_j = g_l \mu_N \left\{ \frac{j(j+1) + l(l+1) - s(s+1)}{2\sqrt{j(j+1)}} \right\}$$

$$+ g_s \mu_N \left\{ \frac{j(j+1) + s(s+1) - l(l+1)}{2\sqrt{j(j+1)}} \right\} \quad \text{... (xiii)}$$

For a spin $\frac{1}{2}$ particle, j can have two values, $j = l \pm \frac{1}{2}$. Thus, for a given j, l one can have the following two values :

For $j = l + \frac{1}{2}$, $l = j - \frac{1}{2}$;

For $j = l - \frac{1}{2}$, $l = j + \frac{1}{2}$;

For the above two cases, one obtains two different values of μ_j from equation (xii).

Using equation (xii)

$$g_j = g_l \left[\frac{j(j+1) + l(l+1) - s(s+1)}{2j(j+1)} \right]$$

$$+ g_s \left[\frac{j(j+1)+s(s+1)-l(l+1)}{2j(j+1)} \right] \quad \ldots (xiv)$$

ıoment, i.e.,

As stated eartier, the measured magnetic moment μ_z is the largest possible component of μ_j in the magnetic field direction and its given by equation (v). Now, replacing I and j, one obtains

$$\mu_z = g_j \, j \, \mu_N$$

$$= \left\{ g_l \frac{\left[j(j+1)+l(l+1)-s(s+1) \right]}{2\left[j(j+1) \right]} + g_s \frac{\left[j(j+1)+s(s+1)-l(l+1) \right]}{2\left[j(j+1) \right]} \right\} \mu_N \quad \ldots (xv)$$

ic magnetic

For $l = j - \frac{1}{2}$, we have

he direction

$$\mu_z = \left[g_l \frac{\left(j - \frac{1}{2}\right)}{j} + \frac{g_s}{2j} \right] j \, \mu_N \quad \ldots (xvi)$$

For $l = j + \frac{1}{2}$, we have

$$\mu_z = \left[g_l \left(\frac{j+\frac{3}{2}}{j+1} \right) - \frac{g_s}{2(j+1)} \right] j \, \mu_N \quad \ldots (xvii)$$

For odd A, either the proton number of odd (in the odd-even nucleus) or the neutron number is odd (in the even-odd nucleus). Thus, we have the following possibilities :

Odd proton $\left(g_l = 1, \; g_s = g_p \right)$:

$$l = \left(j - \frac{1}{2} \right) : \; \mu_z = \left[j - \frac{1}{2} + \frac{g_p}{2} \right] \mu_N \quad \ldots (xviii)$$

$$l = \left(j + \frac{1}{2} \right) : \; \mu_z = \left(\frac{j}{j+1} \right) \left[j + \frac{3}{2} - \frac{g_p}{2} \right] \mu_N \quad \ldots (xix)$$

CAREER ENDEAVOUR

Odd neutron $\left(g_l = 0, \; g_s = g_n \right)$:

owing two

$$l = j - \frac{1}{2} : \; \mu_z = \frac{g_n \mu_N}{2} \quad \ldots (xx)$$

$$l = j + \frac{1}{2} : \; \mu_z = -\frac{j}{(j+1)} \frac{g_n \, \mu_N}{2} \quad \ldots (xxi)$$

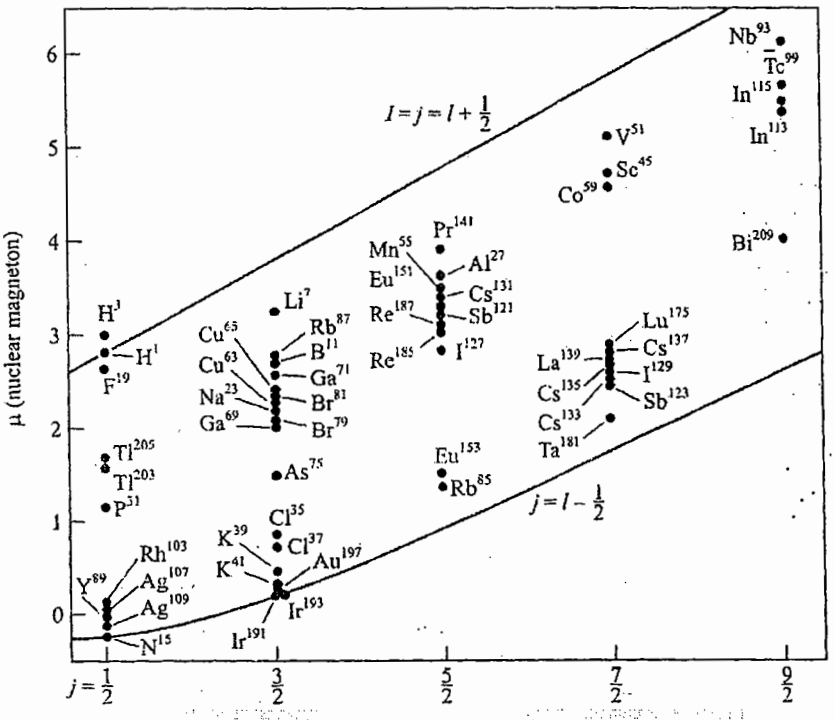


Figure (a) : Magnetic moment versus total angular momentum graph for proton

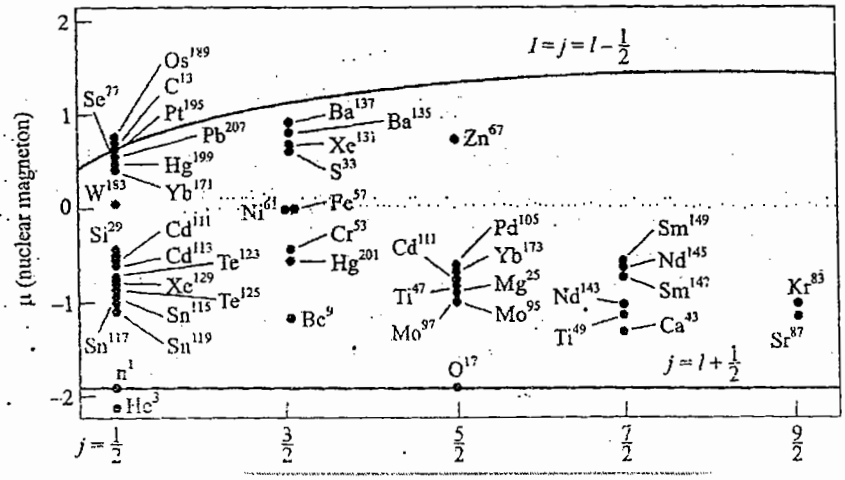

μ (nuclear magneton)
$I=j=l-\frac{1}{2}$
$j=l+\frac{1}{2}$
$j=\frac{1}{2}$
$\frac{3}{2}$
$\frac{5}{2}$
$\frac{7}{2}$
$\frac{9}{2}$
Se⁷⁷ Os¹⁸⁹ C¹³ Pt¹⁹⁵ Pb²⁰⁷ Hg¹⁹⁹ Yb¹⁷¹ W¹⁸³ Si²⁹ Cd¹¹¹ Cd¹¹³ Te¹²³ Xe¹²⁹ Sn¹¹⁵ Te¹²⁵ Sn¹¹⁷ Sn¹¹⁹ n¹ He³
Ba¹³⁷ Ba¹³⁵ Xe¹³¹ S³³ Ni⁶¹ Fe⁵⁷ Cr⁵³ Hg²⁰¹ Be⁹ O¹⁷
Zn⁶⁷ Cd¹¹¹ Ti⁴⁷ Mo⁹⁷ Pd¹⁰⁵ Yb¹⁷³ Mg²⁵ Mo⁹⁵ Nd¹⁴³ Ti⁴⁹
Sm¹⁴⁹ Nd¹⁴⁵ Sm¹⁴⁷ Ca⁴³ Kr⁸³ Sr⁸⁷

Figure (b) : Magnetic moment versus total angular momentum graph for neutron

The values of g_p and g_n are given by equation (iii). We may note that the above values of the nuclear magnetic moments are known as Schmidt limits. These limits are shown joined by lines in figure (a) and (b) together with the measured magnetic moments of odd-A nuclei. In this figure Schmidt values are plotted as functions of $I = J$ for the four cases discussed above. These plots are known as Schmidt diagram.

(3) Determination of electric quadrupole moment of ground state:

If j is the spin of ground state then according to collective model the quadrupole is given by the electric

quadrupole moment = eQ, where $Q = -\frac{3}{5}\left(\frac{2j-1}{2j+2}\right)R^2$ and $R = 1.2\,A^{1/3}$ fm

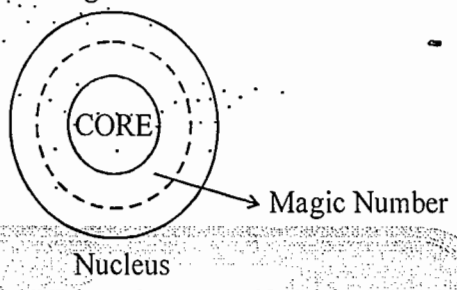
Limitations of the shell model:

1. The model does not predict the correct value of spin quantum number j in certain nuclei, e.g., $^{23}_{11}Na$ where the predicted value is $j = 5/2$ while the correct value is 1/2.
2. The following four stable nuclei $^{2}_{1}H$, $^{6}_{3}Li$, $^{10}_{5}B$ and $^{14}_{7}N$ do not fit into this model.

3. The model cannot explain the observed first excited states in even-even nuclei at energies much lower than those expected from single particle excitation. It also fails to explain the observed large quadrupole moment of odd-A nuclei, in particular of those having A-value far away from magic numbers.

C. Collective model:

- Liquid drop model fails to explain spin, electromagnetic moments, magic number.
- Shell model can not explain deviation of magnetic moment from schmidt limit and large quadrupole moment of certain nuclei and fission and fusion.
- A nucleus consist of a central CORE and some extra nucleons around the CORE. CORE consists of number of nucleons equal to the magic number.



The loose nucleons outside the CORE have large centrifugal force on the CORE. If their number is large, they are able to deform the CORE. The total energy of deformed nucleus can be written as:

$$\boxed{E = E_N + E_V + E_R}$$

E_N – due to the extra nucleons
E_V – Vibrational motion
E_R – Because of rotational motion.

The Rotational Spectrum of Nucleus :

The kinetic energy of a rotating object is $\frac{1}{2}I\omega^2$, where I is the moment of inertia. In terms of the angular momentum $L = I\omega$, the energy is $L^2/2I$. Taking the quantum mechanical value of L^2, and letting J represent the angular momentum quantum number, gives

$$E = \frac{\hbar^2}{2I}J(J+1)$$

For the energies of a rotating object in quantum mechanics. Increasing the quantum number J corresponds to adding rotational energy to the nucleus, and the nuclear excited states form a sequence known as **rotational band.** (Excited states in molecules also form rotational bands, corresponding to rotations of the molecule about is centre of mass.) The ground state of an even-Z, even-N nucleus is always a 0^+ state, and the mirror symmetry of the nucleus restricts the sequence of rotational states in this special case to even values of J. We therefore, expect to see the following sequence of states:

$$E(0^+) = 0$$

$$E(2^+) = 2\times(2+1)\left(\frac{\hbar^2}{2I}\right)$$

$$E(4^+) = 4\times(4+1)\left(\frac{\hbar^2}{2I}\right)$$

$$E\left(6^+\right)=6\times(6+1)\left(\frac{\hbar^2}{2I}\right)$$

$$E\left(8^+\right)=8\times(8+1)\left(\frac{\hbar^2}{2I}\right)$$

and so on.

The first excited state is at $E\left(2^+\right)=91.4\ keV$, and thus we have $\frac{\hbar^2}{2I}=15.2\ keV$. The energies of the next few states in the ground-state rotational band are computed to be

$$E\left(4^+\right)=4\times(4+1)\left(\frac{\hbar^2}{2I}\right)=305\text{ keV}\quad(\text{measured }300\text{ keV})$$

$$E\left(6^+\right)=6\times(6+1)\left(\frac{\hbar^2}{2I}\right)=640\text{ keV}\quad(\text{measured }614\text{ keV})$$

$$E\left(8^+\right)=8\times(8+1)\left(\frac{\hbar^2}{2I}\right)=1097\text{ keV}\quad(\text{measured }1025\text{ keV})$$

The calculated energy levels are notquite exact (perhaps because the nucleus behaves somewhat like a fluid of nucleus and not quite like a rigid object with a fixed moment of inertia), but are good enough to give us confidence that we have at least a rough idea of the origin of the excited levels. In particular, the predicted ratio $E\left(4^+\right)/E\left(2^+\right)$ is 3.33, in remarkable agreement with the systematics of nuclear levels for $150<A<190$ and $A>230$.

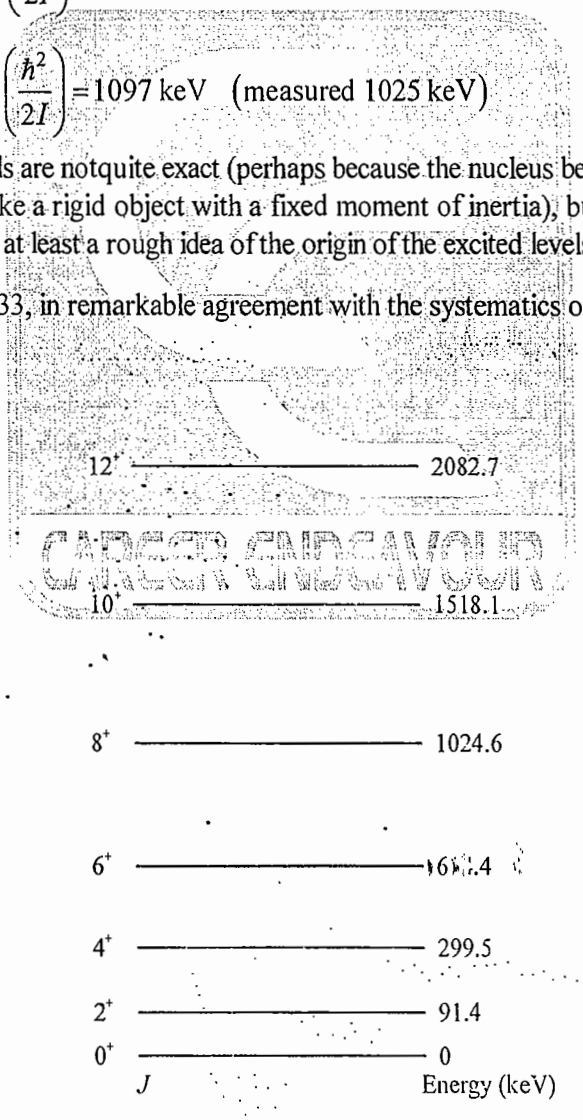


Fig. : The excited states resulting from rotation of the ground state in ^{164}Er.

D. Fermi gas model :

We assume the Fermi gas model in which the particles move without interaction in a sphere of radius R and volume V as the particles in an ideal gas do. According to the theory of the Fermi gas, the number of states of protons with momentum between momenta p and $p+dp$ in a volume V is $\frac{2V\,4\pi p^2 dp}{(2\pi\hbar)^3}$. The spin-weight factor 2 originates from the two spin state of the proton. The ground state of the system will correspond to zero temperature, with momentum spread of zero to maxium p_{F_P} so that the number of protons in this lowest energy state is given by

$$Z = \int_0^{p_{F_P}} \frac{2V 4\pi p^2 dp}{(2\pi\hbar)^3} = \frac{V 8\pi}{h^3}\frac{(p_{F_P})^3}{3}$$

Therefore, the fermi momentum for the protons is

$$p_{F_P} = \hbar K_{F_P} = h\left(\frac{3Z}{8\pi V}\right)^{1/3} = h\left(\frac{3}{8\pi}\rho_P\right)^{1/3}$$

where, $\rho_P = \frac{Z}{V}$ is the proton density.

Similarly, for the neutrons, we have

$$p_{F_N} = \hbar K_{F_N} = h\left(\frac{3N}{8\pi V}\right)^{1/3} = h\left(\frac{3}{8\pi}\rho_P\right)^{1/3}$$

where, $\rho_N = \frac{N}{V}$ is the neutron density. If we define the total nuclear density $\rho_A = \frac{A}{V} = \frac{(N+Z)}{V}$ and assume that there are four particles in each state with equal numbers of neutrons and protons, the fermi momentum is given by

$$p_{F_A} = \hbar k_{F_A} = h\left(\frac{3A}{16\pi V}\right)^{1/3} = h\left(\frac{3}{16\pi}\rho_A\right)^{1/3}$$

The total kinetic energy of the protons in the ground state.

$$T_P = \int_0^{p_{F_P}} \left(2V\frac{4\pi p^2 dp}{h^3}\frac{p^2}{2m}\right) = \frac{8\pi V}{2mh^3}\frac{(p_{F_P})^5}{5} = \frac{4\pi}{5m}\left(\frac{3}{8\pi}\right)^{5/3}\frac{h^2}{V^{2/3}}Z^{5/3}$$

Similarly, the total kinetic energy for neutrons is

$$T_N = \frac{4\pi}{5m}\left(\frac{3}{8\pi}\right)^{5/3}\frac{h^2}{V^{2/3}}N^{5/3}$$

Therefore, the total kinetic energy of N neutrons and Z protons is then,

$$\boxed{K.E. = \frac{4\pi}{5m}\left(\frac{3}{8\pi}\right)^{5/3}\frac{h^2}{V^{2/3}}\left(Z^{5/3}+N^{5/3}\right)}$$

SOLVED PROBLEMS

1. If the masses of $^{40}_{20}Ca$, $^{41}_{20}Ca$ and $^{39}_{20}Ca$ are 39.962589 a.m.u, 40.962275 a.m.u and 38.970691 a.m.u. respectively, calculate the energy difference between $1d_{3/2}$ and $1f_{7/2}$ neutron shells.

Soln. $^{39}_{20}Ca$ has one neutron missing in the $1d_{3/2}$ shell; $^{40}_{20}Ca$ completes this shell and $^{41}_{20}Ca$ adds a neutron to $1f_{7/2}$ shell.

Therefore, B.E. of $1d_{3/2}$ neutron in $^{40}_{20}Ca$ is:

$$B_1 = (M_{39} + M_n - M_{40})c^2 = (38.97069 + 1.008665 - 39.962589) \times 931 \text{ MeV}$$
$$= 0.016766 \times 931 \text{ MeV} = 15.61 \text{ MeV}$$

B.E. of $1f_{7/2}$ neutron in $^{40}_{20}Ca$ is:

$$\therefore B_2 = (M_{40} + M_n - M_{41})c^2$$
$$= (39.962589 + 1.008665 - 40.96225) \times 931 \text{ MeV}$$
$$= 0.008979 \times 931 \text{ MeV} = 8.36 \text{ MeV}$$

Therefore, the energy separation ΔE is the difference $B_1 - B_2$

$$\therefore \quad \Delta E = B_1 - B_2 = 15.61 - 8.36 = 7.25 \text{ MeV}$$

2. Find the total angular momentum and parity for the ground state of $^{33}_{16}S$ nucleus, using the shell model.

Soln. For $^{33}_{16}S$:

$$16P - (1s_{1/2})^2 (1p_{3/2})^4 (1p_{1/2})^2 (1d_{5/2})^6 (2s_{1/2})^2$$
$$17N - (1s_{1/2})^2 (1p_{3/2})^4 (1p_{1/2})^2 (1d_{5/2})^6 (2s_{1/2})^2 (1d_{3/2})^1$$

The total angular momentum or spin of the nucleus $^{33}_{16}S$ is the total angular momentum of the last unpaired neutron.

$$\therefore \quad J = 3/2; \quad \ell = 2 \text{ for } d \text{ state}$$

$$\therefore \quad \text{Parity} = (-1)^2 = +1, \text{ even parity}$$

3. Compute the binding energy of the last proton in a nucleus of ^{12}C if the mass of ^{12}C-nucleus is 12.00052 was a.m.u. and the mass of the ^{11}B-nucleus is 11.01006 a.m.u. The mass of proton is 1.00759 a.m.u.

Soln. On the addition of a proton, the ^{11}B-nucleus is converted into ^{12}C nucleus. The excess of mass of ^{12}C over ^{11}B is:

12.00052 – 11.01006 = 0.99046 a.m.u.

The mass of proton is 1.00759 a.m.u. Thus the proton, when added to the nucleus, suffers mass loss.

Mass loss, $\Delta m = 1.00759 - 0.99046 = 0.01713$ a.m.u.

$\therefore$ Equivalent energy, $\Delta E = 0.01713 \times 931 \simeq 15.95$ MeV

$\therefore$ Binding energy of the last proton = 15.95 MeV

4. Establish the relation $A \simeq 2Z$ for light nuclei using the semi-empirical mass formula, given $a_c = 0.71$ MeV, $a_n = 22.7$ MeV, $M\left(^1_1H\right) = 1.0078$ a.m.u., $M(N) = 1.0086$ a.m.u.

Soln. The mass M of a nucleus of mass number A and charge number Z according to the semi-empirical formula, is given by

$$M = Z\,M_H + (A-Z)M_n - \frac{1}{c^2}\left(a_v A - a_s A^{2/3} - a_c \frac{Z^2}{A^{1/3}} - \frac{a_n (A-2Z)^2}{A} \pm E_\delta\right)$$

For odd A nuclei, we have $E_\delta = 0$

The mass of the most stable nucleus in a family of isobars is given by the condition $(\partial M / \partial Z)_A = 0$.

$$\Rightarrow \quad (\partial M/\partial Z)_A = (M_H - M_n)c^2 + 2a_c \frac{Z}{A^{1/3}} - 4a_n \frac{(A-2Z)}{A} = 0$$

$$\Rightarrow \quad 2a_c \frac{Z}{A^{1/3}} - 4a_n \frac{(A-2Z)}{A} = (M_n - M_H)c^2$$

$$\Rightarrow \quad 2Z\left(\frac{a_c}{A^{1/3}} + \frac{4a_n}{A}\right) = (M_n - M_H)c^2 + 4a_n$$

$$\Rightarrow \quad \frac{2Z}{A}\left(a_c A^{2/3} + 4a_n\right) = (M_n - M_H)c^2 + 4a_n$$

$$\therefore \quad \frac{2Z}{A}\left\{\frac{a_c A^{2/3}}{4a_n} + 1\right\} = \left\{\frac{(M_n - M_H)c^2}{4a_n} + 1\right\}$$

Or, $$Z = \frac{A}{2}\left\{\frac{1 + (M_n - M_H)c^2/4a_n}{1 + \frac{a_c}{4a_n}A^{2/3}}\right\}$$

Now, $a_c = 0.71$ MeV, $a_n = 22.7$ MeV. $\quad \therefore\ a_c/4a_n = 0.0078$ and $(M_n - M_H)c^2/4a_n = 0.0082$

$$\therefore \quad Z = \frac{A}{2}\Big/\left\{\frac{1 + 0.0082}{1 + 0.0078\,A^{2/3}}\right\} \simeq \frac{A}{2}, \text{ for light nuclei.}$$

$\therefore\ A \simeq 2Z$, for light nuclei.

5. Using the semi-empirical binding energy formula, calculate the binding energy of $^{40}_{20}Ca$.

Soln. The semi-empirical binding energy formula is:

$$B.E. = a_v A - a_s A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}} - a_n \frac{(A-2Z)^2}{A} + \delta A^{-3/4}$$

where $a_v = 15.75$ MeV, $a_s = 17.80$ MeV, $a_c = 0.71$ MeV;

$a_n = 22.7$ MeV and $\delta = 34$ as A = even = 40, Z = even = 20

$\therefore \quad a_v A = 15.75 \times 40 = 630$ MeV; $\quad a_s A^{2/3} = 17.80 \times 40^{2/3} = 17.80 \times 11.696 = 208.2$ MeV

$$a_c \frac{Z(Z-1)}{A^{1/3}} = \frac{0.71 \times 20 \times 19}{40^{1/3}} = \frac{0.71 \times 20 \times 19}{3.2} = 84.3 \text{ MeV}$$

$$a_n \frac{(A-2Z)^2}{A} = a_n \times 0 = 0$$

$$\delta A^{-3/4} = 34 \times 40^{-3/4} = 34 \times 0.063 = 2.14 \text{ MeV}$$

$$\therefore \quad B.E. = 630 - [208.2 + 84.3 - 2.14] = 339.64 \text{ MeV}$$

6. Using the semi-empirical binding energy formula, find the atomic number of the most stable nucleus for a given mass number A. Hence explain which is the most stable among ${}^{6}_{2}\text{He}$, ${}^{6}_{4}\text{Be}$ and ${}^{6}_{3}\text{Li}$

Soln. Writing E_b for binding energy, $E_b = a_v A - a_s A^{2/3} - a_c \frac{Z^2}{A^{1/3}} - a_n \frac{(A-2Z)^2}{A} \pm \frac{\delta}{A^{3/4}}$

where, $Z(Z-1) \simeq Z^2$ has been taken.

Now, for most stable nucleus, E_b must be maximum for a given mass number A, i.e.,

$$\left(\frac{\partial E_b}{\partial Z}\right)_{A=const.} = -2a_c A^{-1/3} Z + 4a_n (A-2Z) A^{-1} = 0$$

$$\Rightarrow \quad 4a_n - 8a_n A^{-1} Z = 2a_c A^{-1/3} Z \; ; \; Z\left(4a_n + a_c A^{2/3}\right) = 2a_n A$$

$$\therefore \quad Z = \frac{A}{2 + (a_c / 2a_n) A^{2/3}} = \frac{A}{2 + 0.015\, A^{2/3}}$$

Substituting the values of a_c and a_n.

He, *Be* and *Li* are all light nuclei for which $0.015\, A^{2/3}$ is negligible and $Z = A/2$. This shows that of the three nuclei, ${}^{6}_{3}Li$ is most stable.

7. Show, by way of computation, which nuclei you would expect to be more stable:

${}^{7}_{3}Li$ or ${}^{8}_{3}Li$; ${}^{9}_{4}Be$ or ${}^{10}_{4}Be$

Soln. For a given mass number A, the atomic number Z of the most stable nucleus is:

$$Z = \frac{A}{2 + 0.015\, A^{2/3}}$$

Now, for $A = 7$, $Z = \frac{7}{2 + 0.015 \times 7^{2/3}} = \frac{7}{2 + 0.055} = \frac{7}{2.055} = 3.4$

for $A = 8$; $Z = \frac{8}{2 + 0.15 \times 8^{2/3}} = \frac{8}{2 + 0.060} = \frac{8}{2.060} = 3.88$

Since of the two Z-values, 3.4 is nearer to 3, the ${}^{7}_{3}\text{Li}$ nucleus is more stable.

Again, for $A = 9$, $Z = \frac{9}{2 + 0.015 \times 9^{2/3}} = \frac{9}{2 + 0.065} = \frac{9}{2.065} = 4.36$

for $A = 10$, $Z = \frac{10}{2 + 0.015 \times 10^{2/3}} = \frac{10}{2 + 0.067} = \frac{10}{2.067} = 4.80$

Since the two Z-values, 4.36 is nearer to 4, the ${}^{9}_{4}Be$ nucleus is more stable.

8. Consider a nuclear level corresponding to a closed shell plus a single proton in a state with the angular momentum quantum numbers l and j. Of course $j = \ell \pm 1/2$. Let g_p be the empirical gyromagnetic ratio of the free proton. Compute the gyromagnetic ratio for the level in question, for each of the two cases $j = \ell + 1/2$ and $j = \ell - 1/2$

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Soln. According to the shell model, the total angular momentum of the nucleons in a closed shell is zero, so is the magnetic moment. This means that the magnetic moment and angular momentum of the nucleus are determined by the only proton outside the closed shell.

As, $\mu_j = \mu_\ell + \mu_s,\ g_j j = g_\ell \ell + g_s s$

We have, $g_j j.j = g_\ell \ell.j + g_s s.j$

With $\ell.j = \frac{1}{2}\left(j^2 + \ell^2 - s^2\right) = \frac{1}{2}\left[j(j+1) + \ell(\ell+1) - s(s+1)\right]$

$$s.j = \frac{1}{2}\left(j^2 + s^2 - \ell^2\right) = \frac{1}{2}\left[j(j+1) + s(s+1) - \ell(\ell+1)\right]$$

$$g_j = g_\ell \frac{j(j+1) + \ell(\ell+1) - s(s+1)}{2j(j+1)} + g_s \frac{j(j+1) + s(s+1) - \ell(\ell+1)}{2j(j+1)}$$

For proton, $g_\ell = 1, g_s = g_p$ the gyromagnetic ratio for free proton $(\ell = 0, j = s)$, $s = \frac{1}{2}$. Hence we have

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$$g_j = \begin{cases} \dfrac{2j-1}{2j} + \dfrac{g_p}{2j} & \text{for } j = \ell + 1/2 \\ \dfrac{1}{j+1}\left(j + \dfrac{3}{2} - \dfrac{g_p}{2}\right) & \text{for } j = \ell - 1/2 \end{cases}$$

The simplest model for low-lying states of nuclei with N and Z between 20 and 28 involves only $f_{7/2}$ nucleons.

9. Using this model in previous question, predict the magnetic dipole moments of $^{41}_{20}Ca$ and $^{41}_{21}Sc$.

Soln. ^{41}Ca has a neutron and ^{41}Sc has a proton outside closed shells in state $1f_{7/2}$. As closed shells do not contribute to the nuclear magnetic moment, the latter is determined by the extra-shell nucleons. The nuclear magnetic moment is given by $\mu = g_j \mu_N$

where 'j' is the total angular momentum, μ_N is the nuclear magneton. for a single nucleon in a central field, the g-factor is

$$g = \frac{(2j-1)g_\ell + g_s}{2j} \quad \text{for } j = \ell + \frac{1}{2}$$

$$g = \frac{(2j+1)g_\ell - g_s}{2(j+1)} \quad \text{for } j = \ell - \frac{1}{2}$$

For neutron, $g_\ell = 0, g_s = g_N = -\frac{1.91}{1/2} = -3.82$. As $\ell = 3$ and $j = \frac{7}{2} = 3 + \frac{1}{2}$, we have for ^{41}Ca.

$$\mu\left(^{41}_{20}Ca\right) = -\frac{3.82}{2j} \times j\mu_N = -1.91\ \mu_N$$

For proton, $g_\ell = 1, g_s = g_p = \frac{2.79}{1/2} = 5.58$. As $j = \frac{7}{2} = 3 + \frac{1}{2}$, we have for ^{41}Sc.

$$\mu\left(^{41}_{21}Sc\right) = \frac{(7-1) + 5.58}{7} \times \frac{7}{2}\mu_N = 5.79\ \mu_N$$

10. The band spectrum of U^{238} base on (0^+) ground state. If the energy of $(2)^+$ state is 44.7 KeV. What is the spin and parity of state having energy 525 KeV.

Soln. Since $E_I \propto I(I+1)$, where I = nuclear spin

$$E_I = k\,I(I+1) \qquad \ldots (1)$$

$$E_2 = k\,2(2+1) \quad \Rightarrow \quad E_2 = k\,2\times 3 \qquad \ldots (2)$$

equation (2) divided by (1)

$$\Rightarrow \quad \frac{E_2}{E} = \frac{6}{I(I+1)} \Rightarrow \quad \frac{6}{I(I+1)} = \frac{44.7}{525}$$

$$\Rightarrow \quad I(I+1) = \frac{6\times 525}{44.7} \approx 72 \qquad \Rightarrow I^2 + I - 72 = 0 \quad \Rightarrow \; I^2 + 9I - 8I - 72 = 0$$

$$\Rightarrow \quad I(I+9) - 8(I+9) = 0 \quad \Rightarrow \; (I-8)(I+9) = 0$$

$I = +8$ and $I \neq$ -ve

Or, $I = 8$ and $\boxed{I^P = 8^+}$

11. The rotation energy level of an even-even nucleus are (0^+) ground state, energy of $2^+ = 44$ KeV and higher state energies are 146, 304, 514 KeV respectively for three exited state. Assign spin and parity to these levels.

Soln. (i) $E_I \propto I(I+1) \Rightarrow E_I = kI(I+1)$, where I = nuclear spin

$$E_2 = k\,2\times 3$$

$$E = k\,I(I+1)$$

$$\frac{E_2}{E} = \frac{6}{I(I+1)} \Rightarrow \frac{6}{I(I+1)} = \frac{44}{146} \Rightarrow I(I+1) = \frac{6\times 146}{44} = 6\times 3.3 = 19.8 = 20$$

$$\Rightarrow \quad I^2 + I = 20 \quad \Rightarrow I^2 + I - 20 = 0 \quad \Rightarrow I^2 + 5I - 4I - 20 = 0$$

$$\Rightarrow \quad I(I+5) - 4(I+5) = 0$$

$$\Rightarrow \quad (I-4) = 0 \quad \Rightarrow \boxed{I = 4}$$

(ii) $$\frac{E_2}{E_3} = \frac{I(I+I)}{I(I+1)} \Rightarrow \frac{146}{304} = \frac{4(4+1)}{I(I+1)} \Rightarrow I(I+1) = \frac{20\times 304}{146} = \frac{3040}{73}$$

$$\Rightarrow \quad I^2 + I = 41.64 \approx 42 \quad \Rightarrow \; I^2 + I - 42 = 0 \quad \Rightarrow \; I^2 + 7I - 6I - 42 = 0$$

$$\Rightarrow \quad I(I+7) - 6(I+7) = 0 \quad \Rightarrow \; (I+7)(7-6) = 0 \Rightarrow \boxed{I = 6}$$

(iii) $$\frac{E_3}{E_4} = \frac{I(I+1)}{I(7+1)} \Rightarrow \frac{304}{514} = \frac{6\times 7}{I(I+1)}, \quad \boxed{I^P = 8^+}$$

12. The difference in the coulomb energy between the mirror nuclei $^{49}_{24}Cr$ and $^{49}_{25}Mn$ is 6 MeV. Assuming that the nuclei have a spherically symmetric charge distribution and that e^2 is approximately 1.0 MeV-fm. Find the radius of the $^{49}_{25}Mn$ nucleus.

Soln. Kinetic coulomb energy of uniformly charged sphere of radius R is

$$E_C = \frac{3}{5}\frac{Z^2e^2}{R} \text{ (in c.g.s units)}, \quad E_C = \frac{3}{5}\times\frac{Z^2e^2}{4\pi\varepsilon_0 R} \text{ (in S.I. units)}$$

13. **Soln.**

14.

KeV. What

If we use S.I. units, then $\frac{e^2}{4\pi\varepsilon_0}$ have the value 1 MeV-fm approximately.

If we use, $E_C = \frac{3}{5}\times\frac{Z(Z-1)e^2}{4\pi\varepsilon_0 R}$ then also the result is same almost.

According to the question, $E_{Cr} - E_{Mn} = 6$ MeV

$$\frac{3}{5}\frac{(Z_{Cr}^2 - Z_{Mn}^2)e^2}{R} = 6 \text{ MeV} \Rightarrow R = \frac{3(Z_{Cr}^2 - Z_{Mn}^2)e^2}{6\times 5 \text{ MeV}}$$

= 4.9 fm = 4.9×10^{-15} fm, where $Z_{Cr} = 25$ and $Z_{Mn} = 24$

13. For what elements should stable isobars exist for (a) $A = 97$ and (b) $A = 80$?

Soln. Semi-empirical mass formula for a ${}_ZX^A$-atom is

$${}_zM^A = M_N(A-Z) + M_H Z - a_v A + a_s A^{2/3} + a_c Z(Z-1)A^{-1/3} + a_n(A-2Z)^2 A^{-1} \pm a_p A^{-3/4}$$

Substituting values of constants in terms of MeV, we have

$${}_zM^A = 939.6(A-Z) + 938.8Z - 14A + 13A^{2/3} + 0.60Z(Z-1)A^{-1/3} + 19(A-2Z)^2 A^{-1} \pm 34A^{-3/4}$$

4 KeV and

nd parity to

For a most stable isobar

$$(\partial M/\partial N)_{Z=Z_A} = -0.8 + 0.60(2Z_A - 1)A^{-1/3} - 76(A-2Z_A)A^{-1} = 0$$

or $Z_A = \frac{76.8 + 0.6A^{1/3}}{152 + 1.2A^{2/3}}A$

For odd value of A, there should be only one stable isobar, the neighbouring isobars are unstable and decay to stable isobar. for $A = 97$.

$$Z_A = \frac{76.8 + 0.6\times(97)^{-1/3}}{152 + 1.2\times(97)^{2/3}}(97) = 42.1$$

Hence the most stable isobar is ${}_{42}Mo^{97}$.

By similar calculations for $A = 80$, we get $Z_A = 35.32$. Hence the most stable isobar will be ${}_{35}Br^{80}$. It is the case of Z-odd, N-odd. It has been discussed in theory that for even-A values, we get two parabolas due to the term $\pm a_v A^{-3/4}$. The odd Z-values are associated with upper and even Z-values with the lower. The possible stable isobars with Z-even values are ${}_{34}Se^{80}$ and ${}_{36}Kr^{80}$.

14. In the semi-empirical mass formula, the volume (V), surface (S), coulomb (C), and pairing (P) contributions to the binding energy of a nucleus ${}_Z^A X$ vary with mass number A as **[TIFR 2015]**

(a) $V \propto A$, $S \propto A^{2/3}$, $C \propto A^{-1/3}$, $P \propto A^{-3/4}$

(b) $V \propto A$, $S \propto A^{1/3}$, $C \propto A^{-1/3}$, $P \propto A^{-3/4}$

suming that

(c) $V \propto A$, $S \propto A^{-2/3}$, $C \propto A^{1/3}$, $P \propto A^{-3/4}$

eV-fm. Find

(d) $V \propto A^2$, $S \propto A^{2/3}$, $C \propto A^{-1/3}$, $P \propto A^{-3/4}$

Soln. The semi emperical mass formula for binding energy can be written as:

B.E. = Volume energy (V) + surface energy (S) + Coulomb energy (C) + asymmetry energy (A) + pairing energy (P) ... (i)

$$\text{B.E.} = a_v A - a_s A^{2/3} - \frac{a_c Z(Z-1)}{A^{1/3}} - \frac{a_a (A-2Z)}{A} \pm \frac{\delta}{A^{3/4}} \quad \text{... (ii)}$$

Therefore, from comparison of (i) and (ii), we get

$V = a_v A \Rightarrow \boxed{V \propto A}$

$S = -a_s A^{2/3} \Rightarrow \boxed{S \propto A^{2/3}}$; $C = -\frac{a_c Z(Z-1)}{A^{1/3}} \Rightarrow \boxed{C \propto A^{-1/3}}$

$P = \left(\pm \frac{\delta}{A^{3/4}}\right) \Rightarrow \boxed{P \propto A^{-3/4}}$

Correct option is (a)

15. The mass difference between the pair of mirror nuclei ${}_6C^{11}$ and ${}_5B^{11}$ is given to be ΔMeV/c^2. According to the semi empirical mass formula, the mass difference between the pair of mirror nuclei ${}^{17}_{9}F$ and ${}^{17}_{8}O$ will approximately be (rest mass of proton m_p = 938.27 MeV/c^2 and rest mass of neutron m_n = 939.57 MeV/c^2).

(a) 1.39 ΔMeV/c^2 (b) $(1.39\Delta + 0.5)$MeV/c^2 (c) 1.86 ΔMeV/c^2 (d) $(1.6\,\Delta + 0.78)$MeV/c^2

Soln. The mass of nucleus ${}^{A}_{Z}X$ (from semi-empirical mass formula)

$$m\left({}^{A}_{Z}X\right) = Zm_p + (A-Z)m_n - \text{B.E.}$$

$$= Zm_p + (A-Z)m_n - \left\{a_v A - a_s A^{2/3} - \frac{a_c Z(Z-1)}{A^{1/3}} - a_{sym}\frac{(A-2Z)^2}{A}\right\}$$

For even-odd and odd-even nucleus pairing term is zero.

Here, we have $m\left({}^{11}_{6}C\right) - m\left({}^{11}_{5}B\right) = \Delta meV/c^2$

$$\Rightarrow \left(6m_p + 5m_n - \left(11a_v - 11^{2/3}a_s - \frac{6\times 5}{11^{1/3}}a_c - \frac{a_{asym}}{11}(11-12)^2\right)\right)$$

$$-\left(5m_p + 6m_n - \left\{11a_v - 11^{2/3}a_s - \left(\frac{5\times 4}{11^{1/3}}\right)a_c - \frac{(11-10)^2}{11}a_{asym}\right\}\right) = \Delta$$

$$\Rightarrow \quad m_p - m_n + \frac{10a_c}{11^{1/3}} = \Delta \Rightarrow a_c = \left(\Delta + m_n - m_p\right)\times\frac{11^{1/3}}{10} \quad \text{... (i)}$$

So, $$m\left({}^{17}_{9}F\right) - m\left({}^{17}_{8}O\right) = m_p - m_n + \frac{16}{17^{1/3}}a_c = m_p - m_n + \frac{16}{17^{1/3}}\times\frac{11^{1/3}}{10}\left(\Delta + m_n - m_p\right)$$

$$= \left(m_n - m_p\right)\left[\frac{8}{5}\times\left(\frac{11}{17}\right)^{1/3} - 1\right] + \frac{8}{5}\times\left(\frac{11}{17}\right)^{1/3}\Delta = (0.499 + 1.384\Delta)$$

$$\approx (1.39\Delta + 0.5)\text{MeV}/c^2$$

Correct option is (b)

16. In the nuclear shell model the spin - parity of $^{37}_{20}Ca$ is

(a) $\left(\frac{1}{2}\right)^-$ (b) $\left(\frac{3}{2}\right)^-$ (c) $\left(\frac{5}{2}\right)^+$ (d) $\left(\frac{3}{2}\right)^+$

Soln. Therefore, in $^{37}_{20}Ca$; number of protons = 20, and number of neutrons = 19.
So, protons are even and they will not provide any contribution to angular momentum.
So, distribution of neutrons will be $(s_{1/2})^2\ (p_{3/2})^4\ (p_{1/2})^2\ (d_{5/2})^6\ (s_{1/2})^2\ (d_{3/2})^1$

Therefore, $j = \frac{3}{2}$ and $l = 2$. So, spin-parity $= \left(\frac{3}{2}\right)^+$

Correct option is (d)

According

17. In the nuclear shell model the spin-parity of $^{41}_{21}X$ is

(a) $\left(\frac{7}{2}\right)^-$ (b) $\left(\frac{7}{2}\right)^+$ (c) $\left(\frac{5}{2}\right)^+$ (d) $\left(\frac{5}{2}\right)^-$

^{7} and $^{17}_{8}O$

$_n$ = 939.57

MeV / c^2

Soln. Therefore, in $^{41}_{21}X$; number of protons = 21, and number of neutrons = 20.
So, neutrons are even they will not provide any contribution to angular momentum.
So, distribution of protons will be

$(S_{1/2})^2\ (P_{3/2})^4\ (P_{1/2})^2\ (d_{5/2})^6\ (S_{1/2})^2\ (d_{3/2})^4\ (f_{7/2})^1$

Therefore, $j = 7/2$ and $l = 3$

So, spin - parity $= \left(\frac{7}{2}\right)^-$

Correct option is (a)

18. The semi-empirical mass formula for the binding energy of nucleus contains a coloumb interaction energy term. This term depends on the mass number A of the nucleus as

(a) $A^{-1/3}$ (b) $A^{1/3}$ (c) $A^{2/3}$ (d) A

Soln. Coulomb energy $= \frac{a_C Z(Z-1)}{A^{1/3}}$; $E_c \propto A^{-1/3}$

Correct option is (a)

19. In nuclear shell model spin -parity of $^{125}_{51}S_b$ is

(a) $\left(\frac{5}{2}\right)^-$ (b) $\left(\frac{7}{2}\right)^-$ (c) $\left(\frac{7}{2}\right)^+$ (d) $\left(\frac{5}{2}\right)^+$

Soln. In $^{125}_{51}S_b$; number of protons = 51, and number of neutron = 74
So, distribution of protons is ;

$(S_{1/2})^2(P_{3/2})^4(P_{1/2})^2(d_{5/2})^6(S_{1/2})^2(d_{3/2})^4(f_{7/2})^8(P_{3/2})^4(f_{5/2})^6(P_{1/2})^2(g_{9/2})^{10}(g_{7/2})^1$

Therefore, $j = 7/2$ and $l = 4$

So spin parity $= \left(\frac{7}{2}\right)^+$

Correct option is (c)

20. The spin and parity of $^{13}_{6}C$ is given by

(a) $\frac{1}{2}$, even (b) $\frac{1}{2}$, odd (c) $\frac{3}{2}$, even (c) $\frac{3}{2}$, odd

Soln. Therefore, in $^{13}_{6}C$, number of protons = 6, number of neutrons = 7

Therefore, protons are even so they will not provide contribution to angular momentum.

So, distribution of neutron is $(s_{1/2})^2 (p_{3/2})^4 (p_{1/2})^1$

Therefore $j = 1/2$ and $l = 1$

So spin - parity = $\frac{1}{2}$, odd

Correct option is (b)

21. The nucleus $^{27}_{14}$Si decays to its "mirror" nucleus $^{27}_{13}$Al by positron emission. The maximum (kinetic energy) energy of the positron is 3.48 MeV. Assume that the mass difference between the nuclei is due to the Coulomb energy. Assume the nuclei to be uniformly charged spheres of charge Ze and radius R. Assuming the radius is given by $r_0 A^{1/3}$, find the above data to estimate r_0.

(a) 1.5 fm (b) 1.73 fm (c) 1.62 fm (d) 1.58 fm

Soln. $^{27}_{14}\text{Si} \rightarrow {}^{27}_{13}\text{Al} + \beta^+ + \nu$

$$Q = \left[m\left(^{27}_{14}\text{Si}\right) - m\left(^{27}_{13}\text{Al}\right) - 2m_e \right] c^2 \quad \ldots (1)$$

Eq. (1) is Q-value in case of β-decay.

The coulomb energy in case of uniformly charged sphere is

$$E_c = \frac{1}{4\pi\varepsilon_0}\frac{3}{5}\frac{Z^2e^2}{R} = \frac{1}{4\pi\varepsilon_0}\frac{3}{5}\frac{Z^2e^2}{r_0 A^{1/3}} \quad \{\text{as } R = r_0 A^{1/3}\}$$

Given that $\therefore E_e = \left[m\left(^{27}_{14}\text{Si}\right) - m\left(^{27}_{13}\text{Al}\right) \right] c^2$ put in Eq. (1)

So, $Q = E_e - 2m_e c^2 \quad \{\because m_e c^2 = 0.51 \text{ MeV}\}$

$3.48 \text{ MeV} = E_e - 1.01 \text{ MeV}$

$\Rightarrow E_e = 4.49 \Rightarrow E_e \approx 4.5 \text{ MeV}$

So, $E_e = \frac{1}{4\pi\varepsilon_0}\frac{3}{5}\frac{e^2}{(27)^{1/3}}\frac{\{14^2 - 13^2\}}{r_0} = 4.5 \times 10^6 \times 1.6 \times 10^{-19}$

$$r_0 = \frac{9\times10^9 \times 3 \times (14^2 - 13^2) \times 1.6 \times 10^{-19}}{5 \times (27)^{1/3} \times 4.5 \times 10^6}$$

$r_0 = 1.728 \times 10^{-15}$ m

$r_0 \approx 1.73$ fm.

Correct option is (b)

CAREER ENDEAVOUR

22. The nuclear binding energy may be approximated by the empirical expression

$$\text{B.E.} = a_1 A - a_2 A^{2/3} - a_3 Z^2 A^{-1/3} - a_4 (A-2Z)^2 A^{-1}$$

Considering a set of isobaric nuclei, derive a relationship between A and Z for naturally occuring nuclei.

(a) $\dfrac{A}{2+\dfrac{a_3}{2a_4}A^{2/3}}$ (b) $\dfrac{A}{\dfrac{a_3}{2a_4}A^{2/3}}$ (c) $\dfrac{A}{2+\dfrac{a_4}{2a_3}A^{2/3}}$ (d) $\dfrac{A}{2+\dfrac{a_4}{2a_3}A^{1/3}}$

Soln. For isobaric nuclei of the same A and different Z, the stable nuclides should satisfy

$$\frac{\partial(\text{B.E.})}{\partial Z} = -2A^{-1/3}a_3 Z + 4a_4 A^{-1}(A-2Z) = 0,$$

Giving $Z = \dfrac{A}{2+\dfrac{a_3}{2a_4}A^{2/3}}$

Correct option is (a)

23. Assume a uranium nucleus breaks up spontaneously into two roughly equal parts. Estimate the reduction in electrostatic energy of the nuclei. (Assume uniform charge distribution; nuclear radius = $1.2 \times 10^{-13} A^{1/3}$ cm).

(a) 415 MeV (b) 390 MeV (c) 364 MeV (d) 445 MeV

Soln. Uranium nucleus has $Z_0 = 92$, $A_0 = 236$, and radius $R_0 = 1.2 \times 10^{-13} A_0^{1/3}$ cm. When it breaks up into to two roughly equal parts, each part has

$$Z = \frac{1}{2}Z_0,\ A_0 = \frac{1}{2}A_0,\ R = 1.2 \times 10^{-13} A^{1/3} \text{ cm}.$$

The electostatic energy of a sphere of a uniformly distributed charge Q is $\dfrac{1}{4\pi\varepsilon_0}\dfrac{3}{5}\dfrac{Q^2}{R}$, where R is the radius.

Then for uranium fission, the electrostatic energy reduction is

$$\Delta E = \frac{1}{4\pi\varepsilon_0}\frac{3}{5}\left[\frac{(Z_0 e)^2}{R_0} - 2\times\frac{(Ze)^2}{R}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{3\times Z_0^2 e^2}{5}\frac{1}{R_0}\left[1-\frac{1}{2^{2/3}}\right] = 0.222\times\frac{Z_0^2}{R_0}e^2\frac{1}{4\pi\varepsilon_0}$$

$$= \frac{0.222\times 92^2}{1.2\times 10^{-13}\times 236^{1/2}}\times\frac{1}{137}\times 1.97\times 10^{-11} = 364 \text{ MeV}.$$

This reduction is the source of the energy released in uranium fission. However, to calculate the actual energy release, some other factors should also be considered such as the increase of surface energy on fission.

Correct option is (c)

24. What is spin of a nucleus of ^{12}C?

(a) 1/2 (b) 3/2 (c) 0 (d) 5/2

Soln. Here number of protons = 6, and number of neutrons = 6.
Hence, there is no unpaired nucleons and each pair of protons and neutrons make resultant spin angular momentum = 0.

Correct option is (c)

25. Based on liquid drop model, the most stable nucleus for isobars of $A = 75$ has proton number (Z) equal to

[Given : $a_c = 0.7$ MeV and $a_s = 23.0$ MeV]

(a) 28 (b) 33 (c) 35 (d) 37

Soln. According to liquid drop model

$$B.E. = a_v A - a_s, A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}} - a_s \frac{(A-2Z)^2}{A} \pm \frac{\delta}{A^{3/4}} \quad \ldots (1)$$

For most stable isobar, $\frac{\partial B.E.}{\partial Z} = 0$

Taking $Z(Z-1) \approx Z^2$

So, $B.E. = a_v A - a_s, A^{2/3} - \frac{a_c Z^2}{A^{1/3}} - \frac{a_s (A-2Z)^2}{A} \pm \frac{\delta}{A^{3/4}} \quad \ldots (2)$

Differentiating Eq. (2) w.r.t. 'Z'

$$\frac{\partial B.E.}{\partial Z} = -\frac{a_c (2Z)}{A^{1/3}} - \frac{a_s (2)}{A}(A-2Z)(-2)$$

$$0 = -\frac{2a_c Z}{A^{1/3}} + \frac{4a_s}{A}(A-2Z)$$

$$\frac{2a_c Z}{A^{1/3}} + \frac{8a_s Z}{A} = 4a_s$$

$$Z = \frac{4a_s}{\frac{2a_c}{A^{1/3}} + \frac{8a_s}{A}} \Rightarrow Z = \frac{A}{\left(\frac{a_c}{2a_s}\right)A^{2/3} + 2} \quad \ldots (3)$$

Putting the value of a_c, a_s and A in Eq. (3), we get

$$Z = \frac{75}{(0.7/2 \times 23)(75)^{2/3} + 2} \Rightarrow Z = 33.$$

Correct option is (b)

26. The difference in coulomb energy for nuclei with $Z+1$ and Z protons is given by

(a) $\frac{1}{4\pi\varepsilon_0}\frac{3}{5}\frac{Ze^2}{R}$ (b) $\frac{6}{5}\frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{R}$ (c) $\frac{1}{4\pi\varepsilon_0}\frac{2}{5}\frac{Ze^2}{R}$ (d) $\frac{1}{4\pi\varepsilon_0}\frac{4}{5}\frac{Ze^2}{R}$

Soln. We are assuming that nucleus contains uniform charge around his whole volume.

So, coulomb energy $= \frac{1}{4\pi\varepsilon_0}\frac{3}{5}\frac{(Ze)^2}{R}$, and also nuclei with $Z+1$ and Z will have almost same radius 'R'.

So, difference in coulomb energy $= \frac{1}{4\pi\varepsilon_0}\frac{3}{5}\frac{(Z+1)^2 e^2}{R} - \frac{1}{4\pi\varepsilon_0}\frac{3}{5}\frac{Z^2 e^2}{R}$

$$= \frac{1}{4\pi\varepsilon_0}\frac{3}{5}\frac{e^2}{R}\left\{Z^2 + 1 + 2Z - Z^2\right\}$$

Neglecting 1 as compared to 2Z, as formula valid for high 'Z' number.

Difference in coulomb energy $= \frac{1}{4\pi\varepsilon_0}\frac{6}{5}\frac{Ze^2}{R}$.

Correct option is (b)

27. The ground state spin and parity of $^{19}F_9$ nuclei will be

(a) Proton : $J^{\pi} = \left(\frac{5}{2}\right)^{+}$ (b) Neutron : $J^{\pi} = \left(\frac{5}{2}\right)^{-}$

(c) Proton : $J^{\pi} = \left(\frac{3}{2}\right)^{+}$ (d) Neutron : $J^{\pi} = \left(\frac{3}{2}\right)^{-}$

Soln. In $^{19}F_9$, the number of protons = 9 and the number of neutrons = 10.
As number of neutrons are even so there will be no contribution of angular momentum from them as all the neutron will make pairs.
Hence, distribution of proton is

$(s_{1/2})^2 (p_{3/2})^4 (p_{1/2})^2 (d_{5/2})^1$. So, $J = \frac{5}{2}$ and $l = 2$

Hence, parity $(\pi) = (-1)^l$; π = positive
So, spin parity is $(5/2)^+$ and is due to protons.
Correct option is (a)

28. According to the nuclear shell-model, the spin and parity of the ground state of the nucleus $_{19}K^{39}$ would be

(a) $(3/2)^+$ (b) $(1/2)^+$ (c) $(3/2)^+$ (d) $(1/2)^-$

Soln. In $^{39}_{16}K$, the number of protons = 19, and the number of neutrons = 20.
As, number of neutrons are even, so there will be number contribution of angular momentum from them as all the neutron will make pairs.
Hence, distribution of proton is $(s_{1/2})^2 (p_{3/2})^4 (p_{1/2})^2 (d_{5/2})^6 (s_{1/2})^2 (d_{3/2})^3$
In $(d_{3/2})$ shell out of 3 protons, 2 will make pair. So, contribution of angular momentum is only from $(d_{3/2})$ unpaired proton.

So; $j = \frac{3}{2}$, and parity = $(-1)^l = (-1)^2$ = positive.

So, spin parity is $(3/2)^+$
Correct option is (a)

29. Using the liquid drop model, find the expression for the most stable isobar for a given odd A. Find the stable atom with $A = 77$.

$\{a_c = 0.58$ MeV and $a_a = 19.3$ MeV$\}$

Soln. For odd A, the pair term in the semiempirical mass formula is zero and the binding energy is given by

$$B.E. = a_v A - a_s A^{2/3} - a_c Z^2 A^{-1/3} - a_a (A-2Z)^2 A^{-1}.$$

For most stable isobar (A is constant) is the one with the maximum binding energy. Thus

$$\frac{d(\text{B.E.})}{dZ} = -2a_c Z A^{-1/3} + 4a_a (A - 2Z) A^{-1} = 0$$

or $$Z = \frac{4a_a}{2a_c A^{-1/3} + 8a_a A^{-1}} = \frac{A}{(a_c/2a_a)A^{2/3} + 2}$$

Using $a_c = 0.58$ MeV and $a_a = 19.3$ MeV, we get,

$$Z = \frac{A}{0.015\, A^{2/3} + 2}.$$

For $A = 77$, we get

$$Z = \frac{77}{(0.015)(77)^{2/3} + 2} = 33.9.$$

PRACTICE SET

1. According to the shell model of the nucleus
   (a) magic numbers exist.
   (b) nucleons interact with their nearest neighbours only.
   (c) nucleons in a nucleus interact with a general force field.
   (d) large electron quadrupole moment exists for certain nuclei.

2. The nuclear spins of ${}_6C^{14}$ and ${}_{12}Mg^{25}$ nuclei and respectively:
   (a) zero and half-integer (b) half-integer and zero
   (c) an integer and half-integer (d) both half-integers

3. The spin and parity of ${}_4Be^9$ nucleus, as predicted by the shell model, are respectively:
   (a) 3/2 and odd (b) 1/2 and odd (c) 3/2 and even (d) 1/2 and even

4. The expression of a_c is:

   (a) $\frac{3e^2}{4\pi \in_0 R_0}$ (b) $\frac{3e^2}{10\pi \in_0 R_0}$ (c) $\frac{3e^2}{20\pi \in_0 R_0}$ (d) $\frac{e^2}{20\pi \in_0 R_0}$

5. The binding energy of a light nucleus (Z, A) in MeV is given by the approximate formula

$$B(A,Z) \approx 16A - 20A^{2/3} - \frac{3}{4}Z^2A^{-1/3} + 30\frac{(N-Z)^2}{A}$$

   where $N = A - Z$ is the neutron number. The value of Z of the most stable isobar for a given A is

   (a) $\frac{A}{2}\left(1-\frac{A^{2/3}}{160}\right)^{-1}$ (b) $\frac{A}{2}$ (c) $\frac{A}{2}\left(1-\frac{A^{2/3}}{120}\right)^{-1}$ (d) $\frac{A}{2}\left(1+\frac{A^{4/3}}{64}\right)$

6. For a nucleus to posses quadrupole moment
   (a) $I = 0$ (b) $I = \frac{1}{2}$ (c) $I > \frac{1}{2}$ (d) spherical shape of nucleus
   I = Nuclear spin.

ANSWER KEY						
Questions	1	2	3	4	5	6
Option	(a)	(a)	(a)	(c)	(b)	(c)

Chapter 3

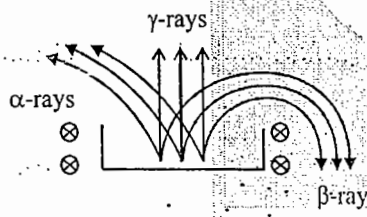
Radioactivity

Introduction:

In 1903, the M. Curie and P. Curie were awarded jointly with Becquerel, the Nobel Prize in physics for his discovery of radioactivity phenomenon. In this chapter, we shall discuss radioactive decay laws of α, β and γ-rays.

Radioactivity:

Radioactivity is the phenomenon of spontaneous distintegration, attended with emission of corpuscular or electromagnetic radiations, of heavy atomic nuclei $\left(\text{for } \frac{N}{Z} > 1.5\right)$ like Uranium, Radium etc. at a constant rate unaffected by any physical or chemical changes or influences such as temperature, pressure etc. to which the atom may be subjected. It is a nuclear property of the active element and in all radioactive processes a transmutation of the element occurs and altogether new nucleus is formed.



CAREER ENDEAVOUR

Radioactive Decay law:

1. On emission of α or β rays, which is usually but not invariably accompanied by γ-ray emission, the emitting parent nuclei transforms into a new daughter element. The daughter element again is radioactive so that the process of successive disintegration continuous till the original active parent nuclei gets transformed into stable one.

2. The rate of radioactive disintegration, that is, the number of atoms that break up at any instant of time t is directly proportional to the number N_t of active nuclei present in the sample under study at that instant.

Decay equation:

Let N_t be the number of active nucleides present in the sample at any time t then we have experimentally,

$$-\frac{dN_t}{dt} \propto N_t \quad \Rightarrow \quad \frac{dN_t}{dt} = -\lambda N_t$$

Where, λ, the constant of proportionality, is known as the decay constant – a characteristic constant of the element (nuclide). The negative sign hints at the fact that N_t decreases with t.

$$\frac{dN_t}{N_t} = -\lambda dt \; ; \; \ell n\, N_t = -\lambda t + A$$

where, A is the constant of integration.

At $t = 0$, $N_t = N_0$ (the initial number of nuclides) and $A = \ell n\, N_0$

Therefore, $$\ell n\left(\frac{N_t}{N_0}\right) = -\lambda t$$

Or, $$\boxed{N_t = N_0 e^{-\lambda t}}$$

Half-life:

The half-life of a radioactive nuclide is defined as the time $T_{½}$ in which the original amount of of radioactive atoms is reduced by way of distintegration to half its value.

$$\therefore \text{ At } t = T_{½},\ N_t = N_0/2;\ \frac{N_0}{2} = N_0 e^{-\lambda T_{½}}$$

$$\therefore T_{½} = \frac{\ell n 2}{\lambda} = \frac{2.303 \times \log 2}{\lambda} = \frac{0.693}{\lambda}$$

Or, $$\boxed{\lambda T_{½} = 0.693 = \text{constant}}$$

Average life:

The average or mean life time $\bar{T}$ of a radioelement is the average life time of all the atoms in the given sample and is defined as the ratio of the total life time of all the atoms to be total number of atoms.

$$\therefore \bar{T} = \frac{t_1 dN_1 + t_2 dN_2 + ...}{dN_1 + dN_2 + ...} = \frac{\Sigma t dN}{\Sigma dN} = \frac{\int_{t=0}^{\infty} t dN}{\int_{N_0}^{0} dN} = \frac{\int_{t=0}^{\infty} t dN}{-N_0}$$

But, we have,

$$dN = d\left(N_0 e^{-\lambda t}\right) = -\lambda N_0 e^{-\lambda t} dt$$

$$\therefore \bar{T} = \lambda \int_0^{\infty} t e^{-\lambda t} dt = \lambda \left[-\frac{t}{\lambda} e^{-\lambda t} + \frac{1}{\lambda} \int_0^{\infty} e^{-\lambda t} dt \right] = \lambda \left[-\frac{t}{\lambda} e^{-\lambda t} - \frac{1}{\lambda^2} e^{-\lambda t} \right]_0^{\infty} = \frac{1}{\lambda}$$

Therefore, $$\boxed{\bar{T} = \frac{1}{\lambda}}$$

We can write also, $$\boxed{T_{½} = \frac{0.693}{\lambda} = 0.693\bar{T}}$$

Activity:

The activity or strength A_t of a radioactive sample at any instant t is defined as the number of disintegration occurring in the sample in unit time at t, that is,

Activity, $$\boxed{A_t = \left|\frac{dN_t}{dt}\right| = \lambda N_t = \frac{0.693}{T} N_t}$$

Units of activity:

The customer unit of radioactivity is called the curie (Ci). It is defined as the activity of any radioactive substance that disintegrates at the rate of 3.7×10^{10} disintegration per second.

$$1\ \text{Ci} = 1\ \text{curie} = 3.7 \times 10^{10}\ \text{disint/sec}$$

$$1\ \text{mCi} = 10^{-3}\ \text{curie}$$

$$1\ \mu\text{Ci} = 10^{-6}\ \text{curie}$$

CAREER ENDEAVOUR

Radioactive displacement law:

1. The α particles have been identifies as Helium nuclei. Suppose we have parent nucleus having mass number A and atomic number Z, after α-decay the parent nucleus become a new daughter element whose atomic number will be $Z-2$ and mass number will be $A-4$. Its position in periodic table will be shifted towards left by two.

$$_{Z}X^{A} \xrightarrow{\alpha} {}_{Z-2}Y^{A-4}$$

2. The β-particle defined as $_{-1}e^{0}$ due to β- emission the parent elements $\left(_{Z}X^{A}\right)$ become a daughter element $\left(_{Z+1}X^{A}\right)$. So, it position in periodic table will be shifted towards right by one.

$$_{Z}X^{A} \xrightarrow{\beta} {}_{Z+1}Y^{A}$$

Natural radioactive decay series:

There are three naturally occuring radioactive series. These three series are

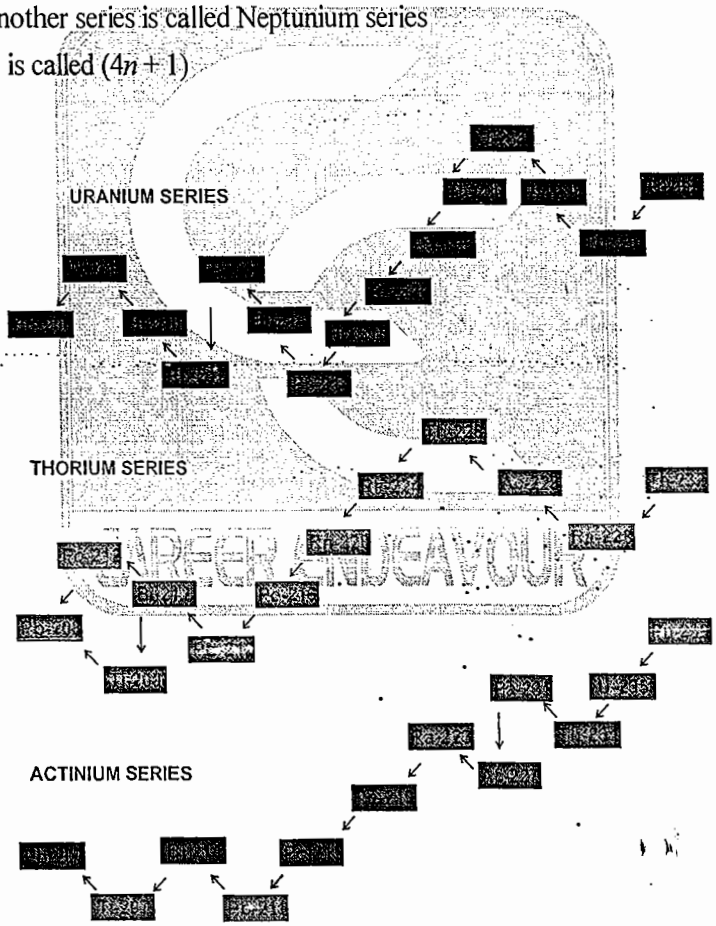
(i) Uranium series is called $(4n+2)$

(ii) Actinium series is called $(4n+3)$

(iii) Thorium series is called $(4n)$

Also there have another series is called Neptunium series

(iv) Neptunium series is called $(4n+1)$



Note: $\nwarrow$ – $(\beta$-decay$)$, $\swarrow$ – $(\alpha$-decay$)$

Successive transformation:

Consider a radioactive nuclide P, symbolised by script 1, to decay into another radioactive nuclide Q (script 2); the latter again decays into a stable end-product R (script 3). For instance,

$$P(1) \xrightarrow{\lambda_1} Q(2) \xrightarrow{\lambda_2} R(3)\ [\text{stable}]$$

Let λ_1, λ_2 be the decay constants of nuclides 1 and 2 respectively, and N_1, N_2, N_3 be the number of atoms of the three kinds at any intant t.

Now the second nuclide would be formed at the rate $\lambda_1 N_1$ by the decay of the parent atom and disappear by its own decay at the rate $\lambda_2 N_2$; the atoms of the end-product i.e. R appear at the rate $\lambda_2 N_2$ by the decay of nuclide 2, but being stable do not disappear. So,

$$\frac{dN_1}{dt} = -\lambda_1 N_1 \quad \text{... (i)}$$

$$\frac{dN_2}{dt} = \lambda_1 N_1 - \lambda_2 N_2 \quad \text{... (ii)}$$

$$\frac{dN_3}{dt} = \lambda_2 N_2 \quad \text{... (iii)}$$

Solving equation (i) : $(N_1)_t = N_{10} e^{-\lambda_1 t}$ ... (iv)

where N_{10} is the number of atoms of nuclide 1, at time $t = 0$

From equation (ii), using (iii), we obtain

$$\frac{dN_2}{dt} = \lambda_1 N_{10}\, e^{-\lambda_1 t} - \lambda_2 N_2$$

Or, $$\frac{dN_2}{dt} + \lambda_2 N_2 = \lambda_1 N_{10} e^{-\lambda_1 t} \quad \text{... (iv_a)}$$

Multiplying both sides of equation (iv-a) by $e^{\lambda_2 t}$, the integrating factor,

$$e^{\lambda_2 t}\left(\frac{dN_2}{dt} + \lambda_2 N_2\right) = \lambda_1 N_{10}\, e^{(\lambda_2 - \lambda_1)t}$$

Or, $$\frac{d}{dt}\left(N_2 e^{\lambda_2 t}\right) = \lambda_1 N_{10} e^{(\lambda_2 - \lambda_1)t}$$

Integrating,

$$N_2 e^{\lambda_2 t} = \frac{\lambda_1}{\lambda_2 - \lambda_1} N_{10} e^{(\lambda_2 - \lambda_1)t} + A \quad \text{... (v)}$$

Where A is the constant of integration.

Now at $t = 0, (N_2)_t = N_{20} = a$ constant. So, from equation (v)

$$A = N_{20} - \frac{\lambda_1}{\lambda_2 - \lambda_1} N_{10}$$

$$\therefore \quad (N_2)_t = \frac{\lambda_1}{\lambda_2 - \lambda_1} N_{10}\left(e^{-\lambda_1 t} - e^{-\lambda_2 t}\right) + N_{20} e^{-\lambda_2 t} \quad \text{... (vi)}$$

From equation (iii), using (vi), we obtain on integration,

$$(N_3)_t = \left(\frac{\lambda_1}{\lambda_2 - \lambda_1} N_{10} - N_{20}\right) e^{-\lambda_2 t} - \frac{\lambda_2}{\lambda_2 - \lambda_1} N_{10}\, e^{-\lambda_1 t} + B \quad \text{... (vii)}$$

At $t = 0, (N_3)_t = N_{30}$. So, from equation (vii), we get

$$B = N_{30} + N_{20} + N_{10}$$

$$\therefore \quad (N_3)_t = N_{30} + N_{20}\left(1 - e^{-\lambda_2 t}\right) + N_{10}\left(1 + \frac{\lambda_1}{\lambda_2 - \lambda_1} e^{-\lambda_2 t} - \frac{\lambda_2}{\lambda_2 - \lambda_1} e^{-\lambda_1 t}\right) \quad \text{... (viii)}$$

l disappear

N_2 by the

Equations (vi), (ii) and (viii) therefore, constitute the solution of the problem.

In most cases, only the first nuclide is present at $t = 0$. So, $N_{20} = N_{30} = 0$. We thus have from (vi) and (vii).

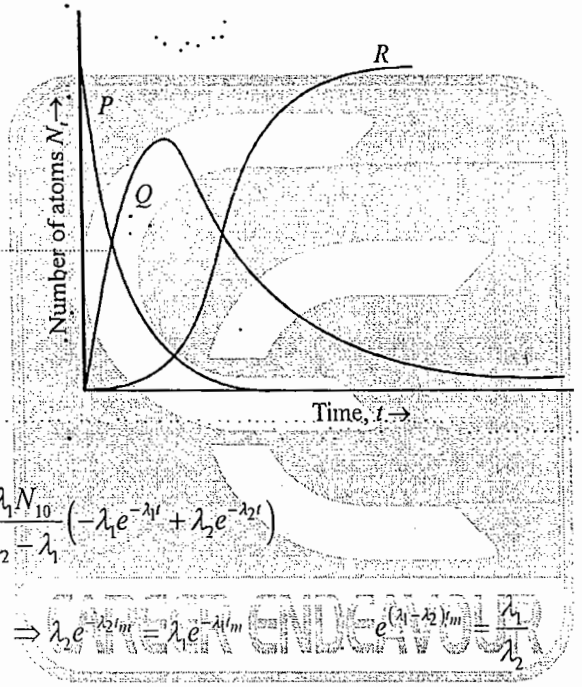
$$(N_2)_t = \frac{\lambda_1}{\lambda_2 - \lambda_1} N_{10} \left(e^{-\lambda_1 t} - e^{-\lambda_2 t} \right) \quad \text{... (ix)}$$

$$(N_3)_t = N_{10} \left(1 + \frac{\lambda_1}{\lambda_2 - \lambda_1} e^{-\lambda_2 t} - \frac{\lambda_2}{\lambda_2 - \lambda_1} e^{-\lambda_1 t} \right) \quad \text{... (x)}$$

The decay of the first nuclide P, the growth and decay of the second nuclide Q and the growth of the third nuclide R are shown in the figure below .

Time for Q(2) to attain a maximum:

Equation (ix) gives how N_2 varies with t. It shows that $N_2 = 0$ at $t = 0$. It increases with increasing t and attains a maximum at $t = t_m$, say.



Now, we have, $\frac{dN_2}{dt} = \frac{\lambda_1 N_{10}}{\lambda_2 - \lambda_1} \left(-\lambda_1 e^{-\lambda_1 t} + \lambda_2 e^{-\lambda_2 t} \right)$

$\therefore \quad \frac{dN_2}{dt} = 0 \Rightarrow \lambda_2 e^{-\lambda_2 t_m} = \lambda_1 e^{-\lambda_1 t_m} \Rightarrow e^{(\lambda_1 - \lambda_2) t_m} = \frac{\lambda_1}{\lambda_2}$

Or, $\quad (\lambda_1 - \lambda_2) t_m = \ell n \frac{\lambda_1}{\lambda_2}$

$\therefore \quad t_m = \frac{1}{\lambda_1 - \lambda_2} \ell n \frac{\lambda_1}{\lambda_2}$ ... (xi)

The t_m corresponds to maximum N_2 can be readily verified by obtaining the second time derivative of N_2 which becomes negative at $t = t_m$.

Branching of radioactivity :

When a radio-element shows that α and β activities, there are definite probabilities of α and β decay constants for the branches. It then be N atom, the total rate of disintegration will be

$$\frac{dN}{dt} = -(\lambda_\alpha + \lambda_\beta) N = \lambda N \qquad \text{where, } \lambda = \lambda_\alpha + \lambda_\beta \text{ the total decay constant.}$$

If $(T_{1/2})_\alpha$, $(T_{1/2})_\beta$ be the partial half lives of α and β decay.

$$\lambda_\alpha = \frac{0.693}{(T_{1/2})_\alpha}, \quad \lambda_\beta = \frac{0.693}{(T_{1/2})_\beta}$$

$$\therefore \quad \lambda = 0.693\left(\frac{1}{(T_{1/2})_\alpha} + \frac{1}{(T_{1/2})_\beta}\right)$$

But if $T_{1/2}$ be the mean half-life of the substance, then

$$\lambda = \frac{0.693}{T_{1/2}}$$

$$\therefore \quad \boxed{\frac{1}{T_{1/2}} = \frac{1}{(T_{1/2})_\alpha} + \frac{1}{(T_{1/2})_\beta}}$$

Determination of constants of radioactivity:

1. For simple decays : We have seen that the activity A of a sample exponentially with time according to the equation,

$$A_t = A_0\, e^{-\lambda t}$$

$$\therefore \quad \ell n\, A_t = \ell n A_0 - \lambda t$$

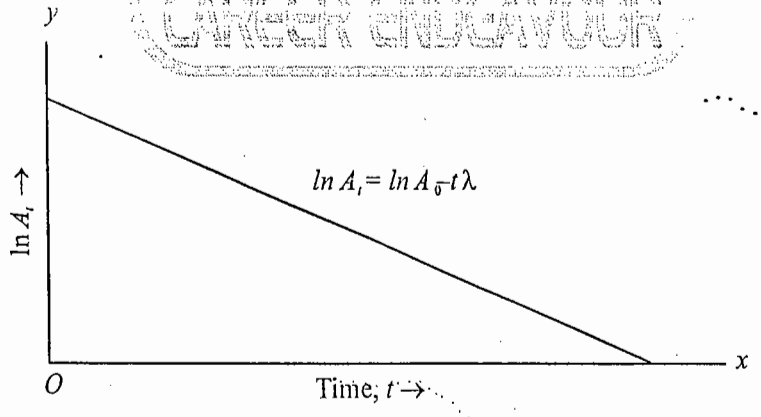
Or, $$y = -\lambda t + C \qquad \ldots (i)$$

where, $y = \ell n\, A_t$ and $C = \ell n\, A_0$

Differentiating (i),

$$\frac{dy}{dt} = -\lambda$$

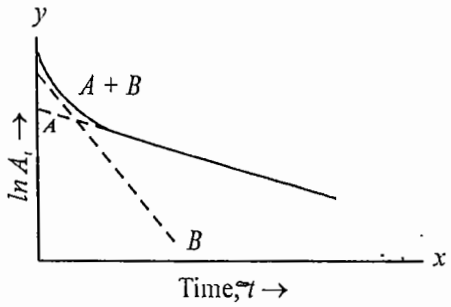
So, when the logarithm of the activity, $ln\, A_t$, is plotted against time t, it will give a straight line with a slope equal to $-\lambda$. This therefore would furnish us a method for evaluating λ, provided we can measure the activity of the sample.



2. Complex decays:

We conclude the section by indicating how one can determine half-lives for complex decays, e.g. two half-lives when a sample contains a mixture of say two activities A and B such that $T_{1/2}(A) >> T_{1/2}(B)$. A graph of the logarithm of the observed activity of the sample is first plotted against time, the observations however are extended over a time large enough compared to $T_{1/2}(B)$. The tail-end of the curve is due to the activity A, since the contribution from B in this region is negligible. So, if the straight part of the graph

is extrapolated to $t = 0$, the contribution to the total observed activity of the sample from A at any instant t can be found. Subtracting this activity of A from the observed total activity at t, we get the activity of B at t. Once the contributions from the two activities are known, their half-lives can be evaluated by the usual procedure.



α-decay:

- **Velocity of α-particles:**

  Let v be the velocity of the α – particles, M be its mass, q the charge, r the radius of the track and B the magnetic field, then $v = Bqr/M$

- **Range of α-particles:**

  The distance through which an α-particle travels in a specified material before stopping to ionise it is called the range of α-particles in that material. The range is thus a sharply defined ionisation path-length. The range depends on (i) the initial energy of the α-particle, (ii) the ionisation potential of the gas and (iii) the chances of collision between the α-particles and on the nature and the temperature and pressure of the gas. With increase of pressure, the range decreases; it increases if the temperature of the gas increased.

  The range R in standard air is proportional to v^3, i.e. $R \propto v^3 \Rightarrow R = av^3$

  The relation is known as the Geiger law, which is valid only in a limited velocity range.

  Since $R \propto v^3$ and the energy $E = \frac{1}{2}mv^2$, the range-energy relationship is

  $$R \propto E^{3/2} \Rightarrow R = bE^{3/2}$$

- **Specific ionisation:**

  The number of ion-pairs formed per unit path-length at any point in the path of the α-particle is called specific ionisation and is symbolised by I.

  Since, $E \propto R^{2/3} \Rightarrow \frac{dE}{dR} \propto R^{-\frac{1}{3}} \propto \frac{1}{v}$

- **Geiger-Nuttal law:**

  An important quantitative relation between the range R of the α-particles and the decay constant λ of the emitting nuclei was experimentally discovered by Geiger and Nuttall (1911) and is called the Geiger-Nutall law. The relation runs as:

  $$\ell n\, \lambda = A + B\, \ell n\, R$$

  where A and B are constants having values different for different radioactive series.

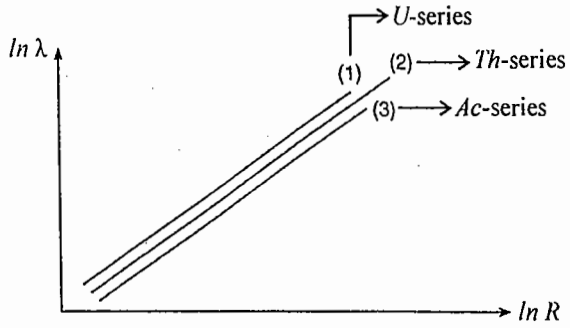


Figure: Variation of $ln\,\lambda$ with $ln\,R$: Geiger-Nuttall law

Since $R \propto E^{3/2} \quad \Rightarrow \ell n\,\lambda = C + D\,\ell n\,E$
where C, D are two constants.

- **α-disintegration energy:**

$$_{Z}X^{A} \rightarrow {}_{Z-2}Y^{A-4} + {}_{2}He^{4}$$

The Q-value of the decay process is known as the α-disintegration energy which is the total energy released in the disintegration process and is given by $Q_\alpha = (M_X - M_\alpha - M_Y)c^2$
where M's are the masses of the particles and c the velocity of light in vacuum.

For heavy nuclei, Q_α is positive, so the decay can occur spontaneously as it does. According to laws of conservation of momentum and energy.

$$0 = M_\alpha v_\alpha - M_Y v_Y$$

and $$Q_\alpha = \frac{1}{2}M_\alpha v_\alpha^2 + \frac{1}{2}M_Y v_Y^2$$

$$\Rightarrow \quad Q_\alpha = \frac{1}{2}M_\alpha v_\alpha^2\left(1+\frac{M_\alpha}{M_Y}\right) = T_\alpha\left(1+\frac{M_\alpha}{M_Y}\right)$$

Kinetic energy of ejected α-particles, $T_\alpha = \dfrac{Q_\alpha}{1 + M_\alpha / M_Y}$

$$\Rightarrow \quad Q_\alpha = T_\alpha\frac{[My + M_\alpha]}{M_y} = T_\alpha\left[\frac{A}{A-4}\right]$$

$$\Rightarrow \boxed{T_\alpha = Q_\alpha\left[1-\frac{4}{A}\right]} \quad \& \quad T_d = Q_\alpha - T_\alpha = \frac{4Q}{A}$$

Where K.E. of α-particle = T_α

As $A \sim 200 \quad \Rightarrow \quad T_\alpha \sim Q_\alpha$

α - decay through tunnelling:

$$_{Z}X^{A} \longrightarrow {}_{Z-2}Y^{A-4} + 2He^{4}$$

$$R = R_0 A^{1/3}$$

$$V(R)=\frac{2(Z-2)|e|^2}{4\pi\in_0 R},\ v=\sqrt{\frac{2E}{M}}$$

$$T=e^{-G}, G=A(Z-2)E^{-1/2}-B(Z-2)^{1/2}R^{1/2}$$

$$A=1.587\times10^{-6}\ \text{J}^{1/2}$$

$$B=94\times10^{6}\ \text{m}^{-1/2}$$

$$\nu=\frac{v}{2R};\ \lambda=\nu T;\ T_{½}=\frac{\ell n\,2}{\lambda}$$

Example: ${}_{92}U^{238}\longrightarrow{}_{90}Th^{234}+{}_{2}He^{4}$ and the energy of the emitted α-particle is $E=6.72\times10^{-13}$ joule. Determine the half-life time of ${}_{92}\text{U}^{238}$.

Soln. Nuclear radius of parent

$$R=R_0A^{1/3}=(1.4\text{ fm})(238)^{1/3}=8.7\text{fm}=8.7\times10^{-15}\text{m}$$

Atomic number of parent nucleus ${}_{92}U^{238}$, $Z=92$

Tunnelling probability,

$$T=e^{-G}=e^{-\left(1.587\times10^{-6}\ \text{J}^{1/2}\right)(92-2)\left(6.72\times10^{-13}\text{J}\right)+\left(94\times10^{6}\ \text{m}^{-1/2}\right)(92-2)^{1/2}\left(8.7\times10^{-15}\text{m}\right)^{1/2}}=\frac{6}{10^{39}}$$

Out of 10^{39} strikes α-emerges out six times only.

Velocity, $$v=\sqrt{\frac{2E}{m}}=\sqrt{\frac{2\left(6.72\times10^{-13}\text{J}\right)}{4\times1.67\times10^{-27}\text{kg}}}=1.42\times10^{7}\ \text{ms}^{-1}$$

Collision frequency, $$\nu=\frac{v}{2R}=\frac{1.42\times10^{7}\ \text{ms}^{-1}}{2\left(8.7\times10^{-15}\text{m}\right)}=8.2\times10^{20}\text{s}^{-1}$$

Decay constant, $$\lambda=\nu T=\left(8.2\times10^{20}\text{s}^{-1}\right)\left(6\times10^{-39}\right)=4.92\times10^{-18}\text{s}^{-1}$$

Half life $$T_{½}=\frac{\ell n\,2}{\lambda}=\frac{\ell n\,2}{4.92\times10^{-18}\text{s}^{-1}}=1.4\times10^{17}\text{s}$$

$$=\frac{1.4\times10^{17}}{365\times24\times60\times60}\text{years}=4.4\times10^{9}\text{ years}$$

The experimental value of half life is 4.5×10^{9} years. So, we have obtained a rough estimate of half life of ${}_{92}U^{238}$.

β-decay:

Let v be the velocity of a given β-particle, B the magnetic flux density, m the relativistic mass of the β-particle and r the radius of the circular track.

Then $\dfrac{mv^2}{r} = Bev \quad \Rightarrow \quad r = \dfrac{mv}{Be} = \dfrac{m_0}{\sqrt{1-v^2/c^2}} \cdot \dfrac{v}{Be}$

- β-decay, we write:

$${}_ZX^A \rightarrow {}_{Z+1}Y^A + {}_{-1}e^0$$

The disintegration energy in β-decay is:

$$Q_{\beta^-} = \left[M_n(A,Z) - M_n(A,Z+1) - m_e\right]c^2$$

$$= \left[M(A,Z) - Zm_e - M(A,Z+1) + (Z+1)m_e - m_e\right]c^2 \text{ (in terms of atomic mass)}$$

$$= \left[M(A,Z) - M(A,Z+1)\right]c^2$$

where M_n is the nuclear mass, M the atomic mass and m_e the mass of electron.

$Q_{\beta^-} > 0$, if $M(A, Z) > M(A, Z+1)$

implying that β-decay occurs only if the mass of the parent atom is greater than that of the daughter atom.

- **β^+- decay:**

$${}_ZX^A \rightarrow {}_{Z-1}Y^A + {}_{+1}e^0$$

$$\Rightarrow \quad Q_{\beta^+} = \left[M_n(A,Z) - M_n(A,Z-1) - m_e\right]c^2$$

$$= \left[M(A,Z) - Zm_e - M(A,Z-1) + (Z-1)m_e - m_e\right]c^2$$

$$= \left[M(A,Z) - M(A,Z-1) - 2m_e\right]c^2$$

For β^+- decay to occur : $Q_{\beta^+} > 0$, if $M(A,Z) > M(A,Z-1) + 2m_e$

i.e. the mass of the parent atom is greater than the daughter atom at least twice the electronic mass, i.e. 1.02 MeV.

- **Electron capture:**

$${}_ZY^A + {}_{-1}e^0 \rightarrow {}_{Z-1}Y^A$$

Therefore, disintegration energy, $Q_e = \left[M_n(A,Z) + m_e - M_n(A,Z-1)\right]c^2 - B_e$ where B_e is the binding energy of the electron to the orbit.

$$\Rightarrow \quad Q_e = \left[M(A,Z) - Zm_e + m_e - M(A,Z-1) + (Z-1)m_e\right]c^2 - B_e$$

$$= \left[M(A,Z) - M(A,Z-1)\right]c^2 - B_e$$

For electron capture to occur: $Q_e > 0$, if $M(A,Z) > M(A,Z-1) + B_e$

i.e. the mass of the parent atom is greater than that of the daughter atom by at least the binding energy of the electron.

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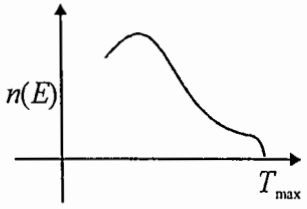
- **Neutrino:**

We see that Q_{β^-} and Q_{β^+} are fixed for a particular parent nucleus. But experimentally it was observed that β-particles came out with varying energy. In the figure given below, the energy spectrum of β-particles is shown.



To explain the continuous energy spectrum of β-particles, Wolfgang Pauli proposed that an additional particle (with zero charge and negligible mass) was also emitted in β-decay. This particle was named neutrino. So, the β-decays are

$$^{A}_{Z}X \longrightarrow {}^{A}_{Z+1}Y + e^- + \bar{\nu}_e$$

$$^{A}_{Z}X \longrightarrow {}^{A}_{Z-1}Y + e^+ + \nu_e$$

and electron capture

$$^{A}_{Z}X + e^- \longrightarrow {}^{A}_{Z-1}Y + \nu_e$$

where, ν_e and $\bar{\nu}_e$ are electrons neutrino and anti-neutrino respectively.

- **Selection rule for β-decay:**

If ℓ_β is odd, initial and final nuclei must have opposite parities (parity changes in these transitions); for even ℓ_β values the initial and final nuclei must have same parity (no change in parity). Furthermore, as in allowed transitions, the emission of leptons (electron and neutrino) in the singlet state (Fermi-selection rule) requires $\Delta I \le \ell_\beta$, whereas triplet-state (GT selection rule) emission requires $\Delta I \le \ell_\beta + 1$. Thus selection rules for forbidden transitions are:

First forbidden – For these transitions $\ell_\beta = 1$ and parity changes.

Fermi-selection rules: $\Delta I = \pm 1, 0 \left(\text{except } 0 \to 0\right)$

Gamow Teller rules: $\Delta I = \pm 2, \pm 1, 0 \left(\text{except } 0 \to 0, \frac{1}{2} \to \frac{1}{2}, 0 \leftrightarrow 1 \right)$

The examples are:

${}^{39}_{18}Ar \rightarrow {}^{39}_{19}K + \beta^-$	$\left(\frac{7^-}{2} \rightarrow \frac{3^+}{2}\right)$	GT
${}^{14}_{6}C \rightarrow {}^{14}_{7}N + \beta^-$	$0 \rightarrow 1$	F
${}^{38}_{17}Cl \rightarrow {}^{38}_{18}Ar + \beta^-$	$\left(2^- \rightarrow 2^+\right)$	$F + GT$ mixed
${}^{139}_{56}Ba \rightarrow {}^{139}_{57}La + \beta^-$	$\left(\frac{7^-}{2} \rightarrow \frac{7^+}{2}\right)$	$F + GT$ mixed
${}^{143}_{59}\mathrm{Pr} \rightarrow {}^{143}_{60}Nd + \beta^-$	$\left(\frac{7^+}{2} \rightarrow \frac{7^-}{2}\right)$	$F + GT$ mixed
${}^{147}_{61}Pm \rightarrow {}^{147}_{62}Sm + \beta^-$	$\left(\frac{7^+}{2} \rightarrow \frac{7^-}{2}\right)$	$F + GT$ mixed
${}^{85}_{36}Kr \rightarrow {}^{85}_{37}Rb + \beta^-$	$\left(\frac{9}{2} \rightarrow \frac{5}{2}\right)$	GT
${}^{89}_{38}Sr \rightarrow {}^{89}_{39}Y + \beta^-$	$\left(\frac{5^+}{2} \rightarrow \frac{1^-}{2}\right)$	GT
${}^{137}_{55}Cs \rightarrow {}^{136}_{56}Ba^* + \beta^-$	$\left(\frac{7^+}{2} \rightarrow \frac{11^-}{2}\right)$	GT

Second forbidden. For these transitions $\ell_\beta = 2$ and no change in parity.

Fermi selection rules: $\Delta I = \pm 2, \pm 1$ (except $0 \leftrightarrow 0$)

Gamow Teller rules: $\Delta I = \pm 3, \pm 2$ (except $0 \leftrightarrow 2$)

The examples are:

${}^{137}_{55}Cs \rightarrow {}^{137}_{56}Ba + \beta^-$	$\left(\frac{7^+}{2} \rightarrow \frac{3^+}{2}\right)$	$F + GT$ mixed
${}^{10}_{4}Be \rightarrow {}^{10}_{5}B + \beta^-$	$\left(0^+ \rightarrow 3^+\right)$	GT
${}^{22}_{11}Na \rightarrow {}^{22}_{10}Ne + \beta^+$	$\left(3^+ \rightarrow 0^+\right)$	GT

- **γ-decay:**
  In this decay there will be no change in Z and A. γ-ray photons are emitted when the nucleus jumps from an excited state to the lower state (after 10^{-14} sec). It is always not possible that when nucleus jumps from higher energy state to lower γ-photon will be emitted.

  Sometimes the nucleus in higher energy state directly give energy to the atomic e^- and hence an e^- may come out rather than a γ-ray photon. This process is known as internal conversion and the e^- so emitted is called as conversion e^-.

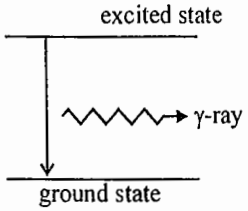
Energy of emitted γ-ray in α-decay:

Let us have a parent nuclei ${}_zX^A$, when it emits a particle it goes to the excited state and it comedown to its ground state by emitting γ-ray.

According energy conservation rule we can write,

$$Q = T_\alpha + T_y + E_y$$

$$E_\gamma = Q_\alpha - \left(T_\alpha + T_y\right) = Q_\alpha - T_\alpha\left(1 + \frac{M_\alpha}{M_y}\right)$$



$$\therefore \quad \boxed{E_\gamma = Q_\alpha - T_\alpha\left(1 + \frac{M_\alpha}{M_y}\right)}$$

• **Selection rule for γ-decay:**

Type	Symbol	Change in Angular momentum, L	Parity change
Electric dipole	E_1	1	Yes
Magnetic dipole	M_1	1	No
Electric quadrupole	E_2	2	No
Magnetic quadrupole	M_2	2	Yes
Electric octupole	E_3	3	Yes
Magnetic octupole	M_3	3	No
Electric 2^L-pole	E_L	L	[No for L even. Yes for odd]
Magnetic 2^L-pole	M_L	L	[Yes for L even. No for L odd]

γ-ray interaction with matter :

There are three fundamental procedure in which γ-ray interact with matter. These three processes

(i) Compton scattering

(ii) Photoelectric effect

(iii) Pair production

The three factors should therefore be combined into one single (total) absorption coefficient, μ in the absorption law. The intensity of the transmitted γ-ray is given by

$$\boxed{I = I_0 e^{-\mu x}}$$ where, x is the thickness of medium.

SOLVED PROBLEMS

1. Radioactive sample has its half-life equal to 60 days. Calculate its (i) disintegration constant, (ii) Its average life, (iii) the time required for 2/3 of the original number of atoms to disintegrate and (iv) the time taken for 1/4 of the original number of atoms to remain unchanged.

Soln. (i) Since $t_{1/2} = 60$ days, $\lambda = 0.693/t_{1/2} = 0.693/60 = 0.01155\ \text{day}^{-1}$

(ii) Since $\lambda = 0.01155\ \text{day}^{-1}$, $\bar{T} = 1/\lambda = 1/0.01155 = 86.58$ days

(iii) Number to be disintegrated $= \frac{2}{3}N_0$. So, the number to remain unchanged $= \frac{1}{3}N_0$

Thus, $N/N_0 = \frac{1}{3}$

Therefore, From the relation $N = N_0 e^{-\lambda t}$, we obtain $\frac{1}{3} = e^{-\lambda t} \Rightarrow \lambda t = \ell n\, 3$

$$\Rightarrow \quad t = \frac{\ell n\, 3}{\lambda} = \frac{2.3026 \times 0.4771}{0.01155} = 95.1 \text{ days}$$

(iv) Here $N / N_0 = \frac{1}{4}$. So, from the decay law, $\frac{1}{4} = e^{-\lambda t} \Rightarrow \lambda t = \ell n 4$

$$\Rightarrow \quad t = \frac{\ell n\, 4}{\lambda} = \frac{2.3026 \times 0.6021}{0.01155} = 120 \text{ days}$$

2. (a) A radioactive substance disintegrates for a time equal to its average life. Calculate the fraction of the original substance disintegrated.
(b) The half-life of a radon is 3.82 days. What fraction of freshly prepared sample of radon will disintegrate in 10 days?

Soln. (a) Here $\bar{T} = t = \frac{1}{\lambda}$ Since, $\frac{N}{N_0} = e^{-\lambda t} = e^{-\lambda \bar{T}} = e^{-1} = 0.368$

Therefore, Fraction disintegrated = 1 – 0.368 = 0.632

(b) Here $T_{1/2} = 3.82$ days and $t = 10$ days. Therefore, we have $\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{3.82} = 0.181 \text{ day}^{-1}$

Now, $\frac{N}{N_0} = e^{-\lambda t} = e^{-0.181 \times 10} = e^{-1.81} = \frac{3}{20}$

$\Rightarrow$ Fraction disintegrated $= 1 - \frac{3}{20} = \frac{17}{20}$

3. The half-life of UX_1 is 24.1 days. How many days, after UX_1 has been isolated, will it take for 90% of it to change to UX_2?

Soln. Here $T_{1/2} = 24.1$ days $\Rightarrow \lambda = 0.693 / 24.1 = 0.0287 \text{ day}^{-1}$

Amount disintegrated = 90/100 $\Rightarrow N / N_0 = 10/100 = 1/10$

Since, $\frac{N}{N_0} = e^{-\lambda t}, \quad \frac{1}{10} = e^{-0.0287 t}$

$$\Rightarrow \quad t = \frac{\ell n\, 10}{0.0287} = \frac{2.3026 \times 1}{0.0287} = 80 \text{ days}$$

4. The half-life of radioactive K-40 is 1.83×10^8 years. Find the number of β-particles emitted per sec per g. of K-40, assuming $\lambda = 1.2 \times 10^{-17}$ s^{-1}, Avogrado number = 6.02×10^{23}.

Soln. Let N_t = Number of atoms of K-40 in 1 g at $t = 6.02 \times 100^{23} / 40$

$$\Rightarrow \quad \left|\frac{dN_t}{dt}\right| = \lambda N_t = \frac{1.2 \times 10^{-17} \times 6.02 \times 10^{23}}{40} = 1.8 \times 10^5$$

So, the number of particles emitted per g of K-40 is 1.8×10^5

5. It is observed that 3.67×10^{10} α-particles are emitted per g of *Ra*-226. Calculate the half-life of *Ra*-226.

Avogadro number = 6.023×10^{23}.

Soln. 1 g of *Ra*-226 = $6.023\times10^{23}/226$ atoms of *Ra*-226. Of these 3.67×10^{10} disintegrate per sec. So, the decay constant.

$$\lambda=\frac{3.67\times10^{10}}{6.023\times10^{23}/226}\ s^{-1} \quad\Rightarrow\quad T_{1/2}=\frac{0.693}{\lambda}=\frac{0.693\times6.023\times10^{23}}{3.67\times10^{10}\times226}s=1595\text{ years}$$

. A large amount of radioactive material of half-life 20 days got spread in a room making the level of radiation 40 times the permissible level of normal occupancy. After how many days would the room be safe for occupation.

oln. Let after t days the room would be safe for occupation. So, in t days, the activity would drop down to 1/40 of its initial value.

$\Rightarrow$ $\frac{N}{N_0}=\frac{1}{40}$. From the relation $\frac{N}{N_0}=e^{-\lambda t}$, $\frac{1}{40}=e^{-(0.693/20)t}$

$\left(\because T_{1/2}=40\text{ days}, \lambda=(0.693/20)\text{ day}^{-1}\right)$ $\Rightarrow$ $\ell n\,40=\frac{0.693}{20}t$

$\Rightarrow$ $t=\frac{2.303\times20}{0.693}\log_{10}40=\frac{2.303\times20\times1.6021}{0.693}=106.4$

Therefore, room would be safe for occupation after 107 days.

Calculate the amount of *Ra*-226 in secular equilibrium with 1 kg of pure *U*-238, given the half-lives of *Ra*-226 and *U*-238 as 1620 years and 4.5×10^9 years respectively.

oln. Let x (in g) be the required amount of *Ra*-226 in secular equilibrium with 1 kg of pure *U*-238. Now:

1 kg of pure *U*-238 = $\frac{6.02\times10^{23}}{238}\times10^3$ atoms of *U*

x g of *Ra*-226 = $\frac{6.02\times10^{23}}{226}\times x$ atoms of *Ra*

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The condition of secular equilibrium gives, $\frac{N_U}{(T_{1/2})_U}=\frac{N_{Ra}}{(T_{1/2})_{Ra}}$

$\Rightarrow$ $\frac{6.02\times10^{23}\times10^3}{238\times4.5\times10^9}=\frac{6.02\times10^{23}\times x}{226\times1620}$ $\Rightarrow$ $x=\frac{226\times1620\times10^3}{238\times4.5\times10^9}=34.18\times10^{-5}\text{ g}=0.34\text{ mg}$

The Half-life of $_{92}U^{238}$ is 4.51×10^9 years. What percentage age of $_{92}U^{238}$ that existed 10^{10} years ago still survives.

ln. $\lambda=\frac{0.693}{t_{1/2}}=\frac{0.693}{4.5\times10^9}$

If N_0 = number of atom of $_{92}U^{238}$ excited 10^{10} year ago $N=N_0$.

Now present, $N=N_oe^{-\lambda t}$ where $t=10^{10}$ years $\Rightarrow$ $\frac{N}{N_0}=e^{\lambda t}$ or $\ell n\frac{N_0}{N}=\lambda t$

$\Rightarrow$ $2.3026\times\log\frac{N_0}{N}=\lambda t=\frac{0.693\times10^{10}}{4.51\times10^9}$ $\Rightarrow$ $\log\frac{N_0}{N}=\frac{0.693\times10}{2.3026\times4.51}=0.6673$

$\Rightarrow \quad \frac{N_0}{N} = \text{anti}\log 0.6673 = 4.648 \qquad \Rightarrow \quad \frac{N}{N_0} = 0.215$

% of $_{92}U^{235}$ now present = 0.215 × 100 = 21.5%

9. The half-life of a radioactive substance is 5 hour. What will be its one third life time?

Soln. $T_{1/2}$ = 5 hrs. $T_{1/2} = \frac{0.693}{\lambda} \quad \Rightarrow \lambda = \frac{0.693}{5} = 0.1386$ per hour also $\frac{N}{N_0} = e^{-\lambda t}$.

In this case $\frac{N}{N_0} = \frac{1}{3} \quad \Rightarrow \frac{1}{3} = e^{-\lambda t} \quad \Rightarrow 3 = e^{\lambda t}$

Hence, $\ell n\, 3 = \lambda t$ or $t = \frac{2.3026 \times \log 3}{0.1386} = 7.93$ hours

10. The activity of certain radio nuclide decreases to 15% of its original value in 10 days. Find its half life.

Soln. Let N_0 be the original number of nuclei and N left behind after 10 days. If λ is the radioactive constant, then

$\Rightarrow \quad \frac{N}{N_0} = e^{-\lambda t} \Rightarrow \frac{15}{100} = e^{-\lambda 10}$

$\Rightarrow \ell n \frac{100}{15} = 10\lambda \Rightarrow \lambda = \frac{1}{10} \ell n \frac{100}{15} = \frac{1}{10} \times 2.3026 \times \log \frac{100}{15} = 0.1897$

$\Rightarrow \quad T_{1/2} = \frac{0.6931}{\lambda} = \frac{0.6931}{0.1897} = 3.65$ day

11. ^{238}U decays with a half life of 4.51×10^9 years, the decay series eventually ending at ^{206}Pb, which is stable. A rock smaple analysis shows that the ratio of the numbers of atoms of ^{206}Pb to ^{238}U is 0.0058. Assuming that all the ^{206}Pb has been produced by the decay of ^{238}U and that all other half lives in the chain are negligible, the age of rock sample is **[JEST 2013]**

(a) 38×10^6 years (b) 48×10^6 years (c) 38×10^7 years (d) 48×10^7 years

Soln. Half life of the ^{238}U is $T_{1/2} = 4.51 \times 10^9$ years

Therefore, decay constant, $N = \frac{0.6931}{4.5 \times 10^9}$ years$^{-1} = 1.54 \times 10^{-10}$ years^{-1} also $\frac{N_{Pb}}{N_{Ur}} = 0.0058$

$\Rightarrow N_{Pb} = 0.0058\, N_{Ur}$ ...(i)

$\therefore$ Total number Uranium atom = $N_{Pb} + N_{Ure}$

(at initial time) = $0.0058\, N_{Ure} + N_{Ure}$

From equation (i)

$\Rightarrow (N_{Ure}) = 1.0058\, N_{Ure}$

$\because \quad N_t = N_e^{-\lambda t}$

$\Rightarrow N_{Ure} = (N_{Ure})_0 e^{-\lambda t} \Rightarrow 1 \times N_{Ure} = 1.0058\, N_{Ure}\, e^{-\lambda t}$

$\Rightarrow \frac{1}{1.0058} = e^{-\lambda t} \Rightarrow -\lambda t = \ell n \left(\frac{1}{1.0058}\right)$

$\Rightarrow t = -\frac{1}{\lambda} \ell n \left(\frac{1}{1.0058}\right) = -\frac{1}{1.54 \times 10^{-10}} \ell n \left(\frac{1}{1.0058}\right)$

$= -\frac{10^{10}}{1.54} \times (-5.78 \times 10^{-3}) = 3.755 \times 10^{10} \times 10^{-3} = 3.75 \times 10^7 \cong 38 \times 10^6$ years.

Correct option is (a)

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12. In the mixture of isotopes normally found on the earth at the present time, $^{238}_{92}U$ has on aboundance of 99.3% and $^{235}_{92}U$ has an aboundance of 0.7%. The measured lifetimes of these isotopes are 6.52×10^9 years and 1.02×10^9 years respectively. Assuming that they were equally aboundant when the earth was formed, the estimated age of the earth, in years is **[JEST 2014]**

(a) 6.0×10^9 (b) 1.0×10^9 (c) 6.0×10^8 (d) 1.0×10^8

Soln. Suppose, the number of $^{235}_{92}U$ nuclei were N (at initial time),

Therefore, the present number at time t is given by $N_{238} = Ne^{-t/T_{\frac{1}{2}}} = Ne^{-t/(6.52\times10^9)}$

Similarly, the number of $^{235}_{92}U$ nulcei at present $N_{235} = Ne^{-t/(1.02\times10^9)}$

Therefore, aboundance of $^{235}_{92}U$ is $0.7\% = \dfrac{0.7}{100} = 7.0\times10^{-3}$

Therefore, according to question, $\dfrac{N_{235}}{(N_{238}+N_{235})} = 7.0\times10^{-3}$

$\Rightarrow \dfrac{N_{235}}{N_{238}} \cong 7.0\times10^{-3}$ ($\because N_{235}$ is negligible in comparison to N_{238})

$\Rightarrow \dfrac{N_{235}}{N_{238}} = \dfrac{Ne^{-t/(1.02\times10^9)}}{Ne^{-t/(6.52\times10^9)}} = 7\times10^{-3} \Rightarrow e^{(-0.9803\times10^{-9})t+(0.15337\times10^{-9})t} = 7\times10^{-3}$

$\Rightarrow e^{(0.827\times10^{-9})t} = \left(\dfrac{1000}{7}\right) \Rightarrow (0.827\times10^{-9})\times t = \dfrac{1000}{7} \Rightarrow t = \dfrac{1}{(0.827\times10^{-9})}\ell n\left(\dfrac{1000}{7}\right)$

$\Rightarrow t = \dfrac{10^9}{0.827} \times 4.96 = 6\times10^9$ years.

Correct option is (a)

13. A detector is used to count the numbr of γ-rays emitted by a radioactive source. If the number of counts recorded in exactly 20 seconds is 10000, the error in the counting rate per second is **[TIFR 2010]**

(a) ± 5.0 (b) ± 22.4 (c) ± 44.7 (d) ± 220.0

Soln. Number of counts in 20 sec = 10000

Therefore, in one second, number of counts $= \dfrac{10000}{20} = 500$

Therefore, rate of detector (r) = 500 counts/sec.

Now, since the detector is counting the number of counts per second. This rate will follow the Poisson distribution.

Here, the rate is 500 counts/sec i.e. the expectation value of the Poisson distribution

$$E(x) = r = 500 \text{ counts/sec}$$

$$\text{error} = \pm\sqrt{\text{variance}} = \pm\sqrt{\text{expectation value}}$$

[Since, for Poisson distribution, variance = expectation value = Parameter (r)]

$$\therefore \quad = \pm\sqrt{r} = \pm\sqrt{500} = \pm 22.5$$

Correct option is (b)

14. A lead container contains 1 gm of a $^{60}_{27}Co$ radioactive source. It is known that a $^{60}_{27}Co$ nucleus emits a β-particle of energy 316 KeV followed by two γ-emissions 1173 and 1333 KeV respectively. Which of the following experimental methods would be the best way to determine the lifetime of the $^{60}_{27}Co$ source?

(a) Measure the change in temperature of the source **[TIFR 2010]**

(b) Measure the weight of the source now and again after one year

(c) Measure the recoil momentum of the nucleus during β-emission

(d) Measure the number of γ-photons emitted by this source

Soln. In order to determine the lifetime of the $^{60}_{27}Co$ source, we need to measure the number of γ-photons emitted by this source. This can be done by mounting the $^{60}_{27}Co$ source in front of a γ-ray detector and count the number of photons in a specific time interval. By repeating this experiment at different times we can determine the half-life of the source.

Correct option is (d).

15. A standard radioactive source is known to decay by emission of γ-rays. The source is provided to a student in a thick sealed capsule of unbreakable plastic and she is asked to find out the half-life. Which of the following would be the most useful advice to the student? **[TIFR 2014]**

(a) The half-life cannot be measured because the initial concentration of the source is not given.

(b) Mount the source in front of a gamma ray detector and count the number of photons detected in one hour.

(c) Measure the mass of the source at different times with an accurate balance having a least count of 1 mg. Plot these values on a curve and fit it with an exponential decay law.

(d) Mount the source in front of a gamma ray detector and count the number of photons detected in a specific time interval. Repeat this experiment at different times and note how the count changes.

Soln. In order to calculate the half-life of the radioactive source, she should mount the source in front of a gamma ray detector and count the number of photons in a specific time interval. By repeating this experiment at different times, she can see how the count of γ-rays changes with time. In this way, she can calculate the half life time of the radioactive source.

Correct option is (d)

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16. Which of the following radioactive decay chains is it possible to observe ? **[TIFR 2015]**

(a) $^{206}_{82}Pb \rightarrow {}^{202}_{80}Hg \rightarrow {}^{202}_{79}Au$ (b) $^{210}_{83}Bi \rightarrow {}^{210}_{84}Po \rightarrow {}^{206}_{82}Pb$

(c) $^{214}_{88}Ra \rightarrow {}^{210}_{86}Rn \rightarrow {}^{207}_{82}Pb$ (d) $^{206}_{82}Pb \rightarrow {}^{202}_{80}Hg \rightarrow {}^{202}_{79}Au$

Soln. In the α-decay, the mass number decreases by 4 and the atomic number decreases by 2 while for β-decay, atomic number increases by 1.

In given options,

(a) $^{206}_{82}Pb \xrightarrow[yes]{\alpha} {}^{202}_{80}Hg \xrightarrow[no]{\beta} {}^{202}_{79}Au$ (b) $^{210}_{83}Bi \xrightarrow[yes]{\beta} {}^{210}_{84}Po \xrightarrow[yes]{\alpha} {}^{206}_{82}Pb$

(c) $^{214}_{88}Ra \xrightarrow[yes]{\alpha} {}^{210}_{86}Rn \xrightarrow[no]{2\alpha} {}^{207}_{82}Pb$ (d) $^{206}_{82}Pb \xrightarrow[yes]{\alpha} {}^{202}_{80}Hg \xrightarrow[no]{\beta} {}^{202}_{79}Au$

Only option (b) satisfies the decay chain.

Correct option is (b)

17. In an experiment, $^{197}_{79}Au$ nuclie were bombared with neutrons leading to formation of $^{198}_{79}Au$, which is unstable. The half-life of $^{198}_{79}Au$ was measured to be 2.25 days and it was found later that this measured half-life was an underestimate by 10%. The corresponding percentage error in the estimated population of $^{198}_{79}Au$ after 9 days is **[TIFR 2015]**

(a) 10% (b) 25% (c) 2.5% (d) 15%

Soln. The measured half-life of $^{128}_{79}Au$, $T_{½} = 2.25$ days

According to question, the measured half-life of $^{128}_{79}Au$ is 10% less than the actual half-life.

Let the actual half-life is x,

According to question, $x - \frac{x}{10} = 2.25 \Rightarrow x = \frac{2.25 \times 10}{9} = 2.5$ days

The amount after $t = 9$ days with actual half-life

$$N(9) = N(0)\ .\ \exp\left(-\frac{0.6931 \times 9}{2.5}\right) = N(0).\exp(-2.5) = 0.08248 \cdot N(0)$$

where, $N(0)$ is today's population of $^{198}_{79}Au$.

The amount after $t = 9$ days with measured half-life

$$N(9) = N(0) \cdot \exp\left(\frac{-0.6931 \times 9}{2.25}\right) = N_0\left(\exp(-2.7724)\right) = 0.0625 \cdot N(0)$$

Therefore, percentage error in the estimation of population

$$= \left(\frac{\text{actual value} - \text{measured value}}{\text{actual value}}\right) \times 100 = \left(\frac{0.08248 \cdot N(0) - 0.0625 \cdot N(0)}{0.08248 \cdot N(0)}\right) \times 100$$

$= 24.22\% \approx 25\%$

Correct option (b)

18. The half-life of a radioactive nuclear source is 9 days. The fraction of nuclei which are left undecayed after 3 days is: **[JEST 2016]**

(a) $\frac{7}{8}$ (b) $\frac{1}{3}$ (c) $\frac{5}{6}$ (d) $\frac{1}{2^{1/3}}$

Soln. Half-life of the radioactive source $(T_{½}) = 9$ days.

Given time $(t) = 3$ days

Therefore, by relation, $N(t) = N(0)\exp\left(\frac{-0.6931 \times t}{T_{½}}\right)$

$$\Rightarrow \quad \frac{N(t)}{N(0)} = \exp\left(\frac{-0.6931 \times 3}{9}\right) = \exp(-0.2310)$$

$$\Rightarrow \quad \boxed{\frac{N(t)}{N(0)} = 0.793739}$$

Out of these four options:

(a) $\frac{7}{8} = 0.875$ (b) $\frac{1}{3} = 0.3333$ (c) $\frac{5}{6} = 0.8333$ (d) $\frac{1}{2^{1/3}} = 0.7937$

OR

Given half-life time = 9 days
Total decay time 3 days.

Therefore, number of half-life = $\frac{3}{9} = \frac{1}{3}$

Therefore, after 3 days the remaining fraction of particle is $\left(\frac{1}{2}\right)^{1/3} = \frac{1}{2^{1/3}}$

Correct option is (d)

19. 1. A sample of radioactive nuclei of certain element can decay only by γ-emission and β-emission. If the half life for γ-emission is 12 minutes, and that for β-emission is 18 minutes. The half-life for the the sample is

(a) 14.4 min (b) 7.2 min (c) 12 min (d) 24 min

Soln. Total rate of decay = (rate of decay by γ-emission) + rate of decay by β-emission)

$$-\lambda N = -\lambda_1 N - \lambda_2 N$$

$$\lambda = \lambda_1 + \lambda_2$$

$$\frac{\ln 2}{t_{1/2}} = \frac{\ln 2}{t_{1/2}(\gamma)} + \frac{\ln 2}{t_{1/2}(\beta)}$$

$$\Rightarrow \frac{1}{t_{1/2}} = \frac{1}{t_{1/2}(\gamma)} + \frac{1}{t_{1/2}(\beta)} \Rightarrow \frac{1}{t_{1/2}} = \frac{1}{12} + \frac{1}{18}$$

$$\frac{1}{t_{1/2}} = \frac{3+2}{36} \Rightarrow t_{1/2} = \frac{36}{5}$$

$$t_{1/2} = 7.2 \text{ min}$$

Correct option is (b)

20. Ten grams of ^{57}Co kept in an open container beta-decays with a half life of 270 days. The weight of the material inside the container after 540 days will be very nearly

(a) 10 g (b) .5 g (c) 2.5 g (d) 1.25 g

Soln. Here $T_{1/2} = 270$ days and $t = 540$ days

Therefore, $t = 2T_{1/2}$ ... (1)

$$N(t) = N_0 e^{-\lambda t}$$

$$N(t) = N_0 e^{-\frac{t \ln 2}{T_{1/2}}}$$

From equation (1);

$$N(t = 2T_{1/2}) = N_o e^{-2T_{1/2}} \frac{\ln 2}{T_{1/2}}$$

$$N(t = 2T_{1/2}) = N_o e^{-\ln 2^2}$$

$$N(t = 2T_{1/2}) = \frac{N_o}{4}$$

Therefore, No of particle will be 1/4 of the initial no of particles
So mass will be also 1/4 of the initial of the initial mass

21.

Soln.

22.

Soln.

$$\text{Mass} = \frac{10g}{4} = 2.5g$$

Correct option is (c)

21. If ^{14}N is at rest, then find the minimum kinetic energy (threshold energy) required to make reaction occur masses of ^{14}N, proton (p), ^{11}B and $^{4}_{2}He$ (in terms of atomic mass unit) are 14.007550 u, 1.008987 u, 11.012811 u and 4.0038794 u (in terms of MeV) is

$$P + {}^{14}N \rightarrow {}^{11}B + {}^{4}_{2}He$$

(a) 0.019 MeV (b) 1.2 MeV (c) 0.15 MeV (d) 0.36 MeV

ion. If the sample is

Soln. Therefore, $\varepsilon_{th}(\text{thershold energy}) = -Q\left(1+\frac{m_A}{m_B}\right)$ ... (1)

Here 'm_A' particle has velocity, which is = m_p and 'm_B' particle is at rest which is = $m(^{14}N)$

Also $Q-value$ = { mass (initial) – mass(final) } c^2

$Q = \{1.008987 + 14.007550 - 11.012811 - 4.003879\}uc^2$

$Q = -0.1425\text{MeV}$

Put value of 'm_A', 'm_B' and Q value in Eq. (1)

$$\varepsilon_{th} = -(-0.1425)\left(1+\frac{1.008987}{14.007550}\right)$$

$\varepsilon_{th} = 0.153MeV$

Correct option is (c)

22. A radioactive element A decays to B with the decay rate λ_A. In turn, B decays to C with the decay rate λ_B. Suppose, initially, only A-type nuclei are present in such a radioactive sample. After what time would the number of B-type nuclei in this sample each its maximum value? **[JNU Ph.D. 2014]**

Soln. $\underset{N_A=N_0}{A} \xrightarrow{\lambda_A} \underset{N_B=0}{B} \xrightarrow{\lambda_B} \underset{N_C=0}{C}$

ght of the

At $t=0$; $N_A = N_0 e^{-\lambda_A t}$

$$\frac{dN_B}{dt} = \lambda_A N_A - N_B\lambda_B$$

$$\Rightarrow \quad \frac{dN_B}{dt} = \lambda_A N_0 e^{-\lambda_A t} - N_B\lambda_B$$

$$\Rightarrow \quad \frac{dN_B}{dt} + (\lambda_B)N_B = (\lambda_A N_0 e^{-\lambda_A t}) \qquad \text{... (1)}$$

By solving equation (1) ; $N_B e^{\lambda_B t} = \int \lambda_A N_0 e^{-\lambda_A t} e^{\lambda_B t} + c$

$$\Rightarrow \quad N_B e^{\lambda_B t} = \lambda_A N_0 \int e^{(\lambda_B - \lambda_A)t} + c$$

$$\Rightarrow \quad N_B e^{\lambda_B t} = \frac{\lambda_A N_0}{(\lambda_B - \lambda_A)} e^{(\lambda_B - \lambda_A)t} + c \qquad \text{... (2)}$$

Using initial condition at $t=0$, $N_B = 0$

$$0 = \frac{\lambda_A N_0}{(\lambda_B - \lambda_A)} + c \Rightarrow c = -\frac{\lambda_A N_0}{(\lambda_B - \lambda_A)}$$

Put in Eq. (2),

So, $$N_B e^{\lambda_B t} = \frac{\lambda_A N_0}{(\lambda_B - \lambda_A)} e^{(\lambda_B - \lambda_A)t} - \frac{\lambda_A N_0}{(\lambda_B - \lambda_A)}$$

$$\Rightarrow \quad N_B e^{\lambda_B t} = \frac{\lambda_A N_0}{(\lambda_B - \lambda_A)} \left\{e^{(\lambda_B - \lambda_A)t} - 1\right\}$$

$$\Rightarrow \quad N_B(t) = \frac{\lambda_A N_0}{\lambda_B - \lambda_A} \left\{e^{-\lambda_A t} - e^{-\lambda_B t}\right\}$$

So, $N_B(t)$ will reach its maximum value when function $y = e^{-\lambda_A t} - e^{-\lambda_B t}$ will maximum so Taking differentiation,

$$\frac{dy}{dt} = -\lambda_A e^{-\lambda_A t} + \lambda_B e^{-\lambda_B t} = 0$$

$$\Rightarrow \quad \lambda_B e^{-\lambda_B t} = \lambda_A e^{-\lambda_A t} \Rightarrow \quad e^{(\lambda_A - \lambda_B)t} = \frac{\lambda_A}{\lambda_B} \Rightarrow \quad (\lambda_A - \lambda_B)t = \ln\left(\frac{\lambda_A}{\lambda_B}\right)$$

$$t_{\max} = \frac{1}{(\lambda_A - \lambda_B)} \ln\left(\frac{\lambda_A}{\lambda_B}\right)$$

So at $t_{\max}$ number of N_B particles will be at maximum.

23. Tritium, the isotropic ^3H, undergoes β-decay with a half-life of 12.5 years, An enriched sample of hydrogen gas containing 0.1 gm of tritium produces 21 calories of heat per hour.

For these data calculate the average energy of the β- particles emitted. [In the starting time]

(a) 4.3 keV (b) 4.3 MeV (c) 8.6 keV (d) 8.6 MeV

Soln. The decay constant is $\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{12.5 \times 365 \times 24} = 6.33 \times 10^{-6}$ hr^{-1}.

Hence, $-\frac{dN}{dt} = \lambda N = \frac{0.1 \times 6.023 \times 10^{23}}{3} = 6.33 \times 10^{-6} = 1.27 \times 10^{17}$

decay per hour and the average energy of the β- particles is

$$E = \frac{21 \times 4.18}{1.27 \times 10^{17}} = 6.91 \times 10^{-16} \text{ J} = 4.3 \text{ keV}.$$

Correct option is (a)

24. You had a sample of 27 gm of radioactive material and you found only 1 gm after 60 minutes. If you had done the measurement 20 minutes earlier, how much material would you have found ?

(a) 3 gm (b) 6 gm (c) 9 gm (d) 18 gm

Soln. Number of particle at $t = 0$ is $27K$, where K is a constant, and number of particle at $t = 60$ min is $= 1K$.

$$\Rightarrow \quad N(t) = N(t=0)e^{-\lambda t}$$

$$K(1) = K(27)e^{-\lambda \times 60}$$

$$e^{60\lambda} = 27$$

$$\Rightarrow \quad (60)\lambda = \ln 27$$

$$\lambda = \frac{3\ln 3}{60}$$

$$\Rightarrow \quad \lambda = \frac{1}{20}\ln 3 \ \text{min}^{-1}$$

And also number of particle at $t = 40\,\text{min}$ is MK, where M is the mass at $t = 40$ minutes.

$$M(t = 40\,\text{min}) = N(t = 0)\,e^{-\lambda t}$$

$$\Rightarrow \quad MK = 27K\,e^{-\left(\frac{1}{20}\ln 3\right)\times 40}$$

$$M = 27\,e^{-2\ln 3}$$

$$M = \frac{27}{e^{\ln 9}} \Rightarrow M = \frac{27}{9}$$

$$M = 3$$

Correct option is (a)

25. If λ_1 and λ_2 are the decay constants for parent and daughter nuclei respectively and $N_1(0)$ is the number of parent nuclei at time $t = 0$, then the number of atoms N_2 of daughter nuclei at time t is given by

(a) $N_2 = \dfrac{N_1(0)\,\lambda_1}{(\lambda_1 - \lambda_2)}\left[e^{-\lambda_2 t} - e^{-\lambda_1 t}\right]$ (b) $N_2 = \dfrac{N_1(0)\,\lambda_1}{(\lambda_1 - \lambda_2)}\left[e^{-\lambda_1 t} - e^{-\lambda_2 t}\right]$

(c) $N_2 = \dfrac{N_1(0)\,\lambda_2}{\lambda_2 - \lambda_1}\left[e^{-\lambda_2 t} - e^{-\lambda_1 t}\right]$ (d) $N_2 = \dfrac{N_1(0)\,\lambda_2}{(\lambda_1 - \lambda_2)}\left[e^{-\lambda_1 t} - e^{-\lambda_2 t}\right]$

ydrogen

Soln. $A \xrightarrow{\lambda_1} B \xrightarrow{\lambda_2} C$

$t = 0 \quad N_1 = N_1(0) \quad N_2 = 0 \quad N_3 = 0$

$\Rightarrow \quad N_1 = N_1(0)\,e^{-\lambda_1 t}$ and $\dfrac{dN_2}{dt} = \lambda_1 N_1 - \lambda_2 N_2$

$$\frac{dN_2}{dt} = \lambda_1 N_1(0)\,e^{-\lambda_1 t} - \lambda_2 N_2$$

$$\frac{dN_2}{dt} + \lambda_2 N_2 = \lambda_1 N_1(0)\,e^{-\lambda_1 t} \qquad \ldots(1)$$

Therefore, Eq. (1) is in form

$$\frac{dy}{dx} + P(x)y = Q(x)$$

had done

So, $N_2 e^{\lambda_2 t} = \int \lambda_1 N_1(0)\,e^{-\lambda_1 t}\,e^{\lambda_2 t} + c$

$$N_2 e^{\lambda_2 t} = \lambda_1 N_1(0)\int e^{t(\lambda_2 - \lambda_1)} + c$$

s = 1 K.

$$\Rightarrow \quad N_2 e^{\lambda_2 t} = \frac{\lambda_1 N_1(0)}{(\lambda_2 - \lambda_1)}\,e^{(\lambda_2 - \lambda_1)t} + c$$

At $t = 0,\ N_2 = 0$

So, $c = -\dfrac{\lambda_1 N_1(0)}{\lambda_2 - \lambda_1}$

$\Rightarrow \quad N_2 e^{\lambda_2 t} = \dfrac{\lambda_1 N_1(0)}{\lambda_2 - \lambda_1}\left[e^{(\lambda_2 - \lambda_1)t} - 1\right]$

$\Rightarrow \quad N_2 = \dfrac{\lambda_1 N_1(0)}{\lambda_2 - \lambda_1}\left[e^{-\lambda_1 t} - e^{-\lambda_2 t}\right] \quad \Rightarrow \quad N_2 = \dfrac{\lambda_1 N_1(0)}{\lambda_1 - \lambda_2}\left[e^{-\lambda_2 t} - e^{-\lambda_1 t}\right]$

Correct option is (c)

26. Calculate the weight in grams of one Curie and one Rutherford of *RaB* (^{214}Pb) from the half-life of 26.8 minutes.

Soln. **(a)** Since, the Curie is equal to 3.7×10^{10} disintegration per sec.

Hence, activity $\left(-\dfrac{dN}{dt}\right) = \lambda N = 3.7 \times 10^{10}$,

or $N = 3.7 \times 10^{10} \times \dfrac{T}{0.693} = 3.7 \times 10^{10} \times \dfrac{26.8 \times 60}{0.693} = 8.5853 \times 10^{13}$ atoms.

From Avogadro's hypothesis 6.02137×10^{26} atoms are contained in 214 kg of *RaB*.

Therefore, 8.5853×10^{13} atoms are contained in $\dfrac{214 \times 8.5853 \times 10^{13}}{6.022137 \times 10^{26}}$ kg $= 3.051 \times 10^{-11}$ kg.

(b) The unit of radioactivity, Rutherford, is equal to 10^6 disintegration per sec.

Hence, activity $\left(-\dfrac{dN}{dt}\right) = \lambda N = 10^6$

or $N = 10^6 \times \dfrac{26.8 \times 60}{0.693} = 2.32 \times 10^9$ atoms.

Using Avogadro's hypothesis, we get

Wt. of one Rutherford $= \dfrac{214 \times 2.32 \times 10^9}{6.022137 \times 10^{26}} = 8.245 \times 10^{-16}$ kg.

27. A piece of an ancient wooden boat shows an activity of ^{14}C of 3.9 disintegrations per minute per gm of carbon. Estimate the age of the boat if the half life of ^{14}C is 5568 years. Assume that the activity of fresh carbon 14 is 15.6 dpm. gm.

Soln. Let the age of the boat be t years.

Number of ^{14}C nuclides at any time t is given by $N = N_0 e^{-\lambda t}$, where N_0 is the total number of nuclides initially and λ its disintegration constant.

Disintegration constant $\lambda = \dfrac{0.693}{T} = \dfrac{0.693}{5568} = 1.245 \times 10^{-4}\ y^{-1}$.

Activity of fresh carbon $= \lambda N_0 = 15.6$.

Activity of an ancient wooden boat $= \lambda N = 3.9$, therefore, $\dfrac{N_0}{N} = \dfrac{15.6}{3.9}$.

Hence, $e^{1.245 \times 10^{-4} t} = \dfrac{15.6}{3.9}$ **or** $1.245 \times 10^{-4} t = 2.3026 \log_{10}\left(\dfrac{15.6}{3.9}\right) = 2.3026 \times 0.6021$

Therefore, $t = 1.118 \times 10^4$ years.

28. If a radioactive material initially contains 3.0 milligrams of uranium ^{234}U. How much it will contain after 1,50,000 years ? What will be its activity at the end of this time ?

[$T = 2.50 \times 10^5$ years, $\lambda = 8.80 \times 10^{-14}$ per sec.]

Soln. Let m be the mass of the uranium ^{234}U remaining after 1,50,000 years.

Therefore, number of atoms in mass $m = N = m \times 6.022137 \times 10^{26}/234$, and number of atoms initially $N_0 = 3 \times 10^{-6} \times (6.022137 \times 10^{26}/234)$.

Substituting the numerical values in equation $N = N_0 e^{-\lambda t}$, we have

$$\frac{m \times 6.022137 \times 10^{26}}{234} = 3 \times 10^{-6} \times \frac{6.022137 \times 10^{26}}{234} e^{-(0.693/2.5 \times 10^5) \times 150000}$$

or $$m = 3 \times 10^{-6} e^{-0.416}$$

$$0.416 = \log_e (3 \times 10^{-6}/m) = 2.3026 \log_{10} (3 \times 10^{-6}/m)$$

or $$m = 3 \times 10^{-6}/\text{Anti}\log_{10} 0.181 = 1.98 \times 10^{-6} \text{ kg.}$$

Activity of uranium at the end of 1,50,000 years $= -\frac{dN}{dt} = \lambda N$.

$$= 8.80 \times 10^{-14} \times 1.98 \times 10^{-6} \times 6.022137 \times 10^{26}/234$$

$$= 4.484 \times 10^5 \text{ disintegrations/sec.}$$

29. Find the number of α-decays that occur in a 1 gm. sample or thorium-232 in one year, if the integration constant λ of thorium-232 is 1.58×10^{-18} sec^{-1}.

Soln. If N_0 is the initial number of nuclei, N the number remaining after time t, then the number of decays during that period $\Delta N = N_0 - N = N_0 - N_0 e^{-\lambda t}$.

In this problem, $\lambda t = 1.58 \times 10^{-18} \times 3.15 \times 10^7 = 4.898 \times 10^{-11}$. This value is very small, hence the exponential term can be expanded as $e^{-\lambda t} = 1 - \lambda t$.

Therefore, $\Delta N = N_0 - N_0(1 - \lambda t) = N_0 \lambda t$.

N_0 the initial number of nuclei in 1 gm $= 6.022137 \times 10^{23}/232 = 2.596 \times 10^{21}$.

Therefore, number of α-decays $\Delta N = 2.596 \times 10^{21} \times 4.896 \times 10^{-11} = 12.98 \times 10^{10}$.

The number is very large but small compared to the initial number of nuclei. This shown that ^{232}Th is the least unstable α-emitter.

30. ^{212}Bi decays to ^{208}Th by α-emission in 34% of the disintegration and to ^{212}Ra by β-emission in 66% of the disintegration. If the total half value period is 60.5 minutes, find the decay constants for α- and β- and total emission.

Soln. Let λ_α and λ_β be the decay constants for α- and β-emission respectively.

For branching decay constant $\lambda_\gamma = \lambda_\alpha + \lambda_\beta$.

In this problem $\lambda_\beta = \left(\frac{66}{100}\right)\lambda_\gamma$, $\lambda_\alpha = \left(\frac{34}{100}\right)\lambda_\gamma$.

Total decay constant is given by

$$\lambda_\gamma = \frac{0.693}{T} = \frac{0.693}{60.5 \times 60} = 1.91 \times 10^{-4}/\text{sec.}$$

$$\therefore \quad \lambda_\beta = 0.66 \times 1.91 \times 10^{-4} = 1.26 \times 10^{-4}/\text{sec.} \quad \text{and} \quad \lambda_\alpha = 0.34 \times 1.91 \times 10^{-4} = 0.65 \times 10^{-4}/\text{sec.}$$

PRACTICE SET

1. Which of the following phenomena can be explained by quantum mechanical tunneling through the barrier?

   (i) α-decay (ii) Field emission of e^{-s} from a metal surface. (iii) β-decay

   Select the correct answer using the code given below:

   (a) i, ii, iii (b) i only (c) i & ii only (d) ii & iii only.

2. ^{10}Be in its first excited state has spin-parity 2^{1}. It gets de-excited to the ground state, which has spin parity 0, by γ-emission. The multipoles carrried by γ are

   (a) $E2$ (b) $M2$ (c) $E2, M2$ (d) $E4$.

3. Which of the following is true for β-decay of the neutron? The process

   (a) Violates both parity and charge conjugation symmetry.
   (b) Violates parity but conserves charge conjugation symmetry.
   (c) Conserves parity but violates charge conjugation symmetry.
   (d) Conserves both parity and charge conjugation symmetry.

4. Consider the alpha-decay reaction, $Po_{84}^{210} \rightarrow Pb_{82}^{206} + He_{2}^{4}$ where atomic masses are

   $m(Po) = 210.0483$ a.m.u.; $m(Pb) = 206.0386$ a.m.u.

   $m(He) = 4.0039$ a.m.u.; 1 a.m.u. $= 931.141$ MeV

   The kinetic energy of the α-particle will be

   (a) 5.4 keV (b) 2.7 keV. (c) 5.4 MeV (d) 10.8 MeV

5. The energy spectrum of β-particles is continuous because

   (a) parity is violated in β-decay (b) β-particles are emitted in the continuum region

   (c) β-decay leads to a three body state (d) β-decay is a weak interaction process

6. The $0 \rightarrow 0$ gamma transition is forbidden because

   (a) photon has integer spin (b) photon is a massless particle

   (c) photon has a definite polarization (d) parity will not be conserved

7. If decay constant of a radioactive sample is λ per minute, then the fraction that decays in the fifth minute is

   (a) $e^{-4\lambda}$ (b) $e^{-5\lambda}$ (c) $e^{-4\lambda} - e^{-5\lambda}$ (d) $e^{-4\lambda} + e^{-5\lambda}$

8. The radioactive decay of a certain material satisfies Poisson statistics with a mean rate of λ per second, what should be the minimum duration of counting (in seconds) so that the relative error is less than 1% ?

   (a) $100/\lambda$ (b) $10^4/\lambda^2$ (c) $10^4/\lambda$ (d) $1/\lambda$

ANSWER KEY					
Questions	1	2	3	4	5
Option	(c)	(a)	(a)	(c)	(c)
Questions	6	7	8		
Option	(d)	(c)	(c)		

Chapter 4

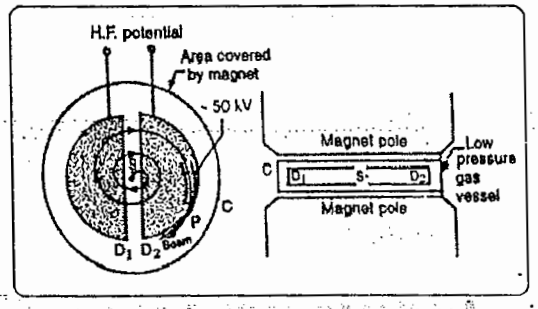
Particle Acceleration

Introduction :

The purpose of an accelerator of chraged particles is to direct against a target a beam of a specific kind of particles of chosen energy. There are many varities of methods for accomplishing this task. All using various arrangement of electric and magnetic field, and this chapter we will read about some types of accelerators.

Cyclotron :

Construction: It consists essentially of two flat semi-circular hollow metal boxes D_1, D_2 which are called the dees (D) because of their shape (figure below). These hollow chambers have their diametric edges parallel and slightly separated from each other so as to produce a narrow gap between them. The dees are placed in an evacuated chamber C and are connected



to an rf-oscillator so that a high frequency (10 MHz) alternating potential is applied between the dees acting as electrodes. The potential between the dees thus alternates rapidly and the electric field in the gap is first directed to one dee and then to the other. The space within each dee is however field-free (zero left). The chamber C is mounted horizontally between the pole pieces of a huge electromagnet that provides an intense and uniform vertical magnetic field of several tesla. Since the field must be uniform over the whole of the dees, the diameter of the poles should be slightly greater than that of the dees.

The ion-source S is located near the mid-point of the gap between the dees. The ions, after being accelerated, are brought out of the chamber throught a window P by a charged deflecting plate to bombard the target.

Principles of action: Let a positive ion of charge q leave the ion-source between the dees and enter the negative dee at the instant with velocity $\vec{v}_0$, normal to the magnetic lines of force. Due to the magnetic field $\vec{B}$, the force acting on the particle is

$$\vec{F} = q\vec{v}_0 \times \vec{B} \qquad \ldots (1)$$

But $\vec{B}$ is perpendicular to the force so that we obtain

$$\vec{F} = qv_0B \qquad \ldots (2)$$

Under the action of the field, the path of the particle is bent into a circle of radius r_0, given by the relation,

$$qv_0B = \frac{mv_0^2}{r_0}$$

$$\therefore \quad \boxed{r_0 = \frac{mv_0}{qB}} \qquad \ldots (3)$$

where m is the mass of the ion (particle).

When the particle is inside the dee, its speed stays constant; but after describing the semicircular path inside the dee, as it leaves it to reach the gap, the electric field of the radio frequency source changes direction synchronously. The particle thus gets a 'kick' and is accelerated into the other dee where, because of its increased speed v, it moves in a semi-circular path of comparatively larger radius r, given by

$$\boxed{r = \frac{mv}{qB}} \qquad \ldots (4)$$

to emerge again into the gap between the dees.

The frequency of revolution of the particle in the circular orbit is given by

$$f = \frac{v}{2\pi r} = \frac{vqB}{2\pi mv} = \left(\frac{q}{m}\right)\frac{B}{2\pi} \qquad \ldots (5)$$

Using equation (4).

The important fact that emerges from equation (5) is that the frequency of revolution is independent of the speed of the ion and the radius of path in the dees. So, if the electric field reverses regularly at a frequency exactly equal to f, the field in the gap is always in the right direction to accelerate a charged particle by an impulse, each time the gap is crossed.

$$\therefore \quad \boxed{\left(\frac{q}{m}\right)\frac{B}{2\pi} = f = f'}$$

This is called the basic resonance equation for a fixed frequency cyclotron, f' being the frequency of the oscillator output.

Thus the revolving charged particle is steadily speeded up describing a flat spiral of increasing radius. Finally, it reaches the periphery of the dees and is brought out of the chamber through window by a negatively charged deflector to impinge on the properly mounted target. We shall now calculate the energy of the particle.

Energy of the particle : The kinetic energy W_k of the charged particle is given by

$$W_k = \frac{1}{2}mv^2$$

Therefore, the maximum kinetic energy of the particles when they leave the cyclotron is

$$W_m = \frac{1}{2}mv_m^2 = \frac{1}{2}m\left(\frac{r_m qB}{m}\right)^2, \text{ using equation (4)}$$

$$= \frac{q^2B^2}{2m}r_m^2 \qquad \ldots (7)$$

Thus, the maximum energy that can be imparted to a particle is directly proportional to the square of (1) the radius of the dees and (2) the magnetic field of the magnet.

The energy of the particles expressed in equivalent voltage V is given by

$$\boxed{V = qB^2r_m^2/2m} \qquad \{\because W_m = qV\} \qquad \ldots (8)$$

Betatron :

As already indicated, the maximum energy that could be imparted to a particle in a cyclotron is limited by the relativistic increase in mass that disturbs the synchronisation. In particular, the cyclotron cannot be used to accelerate electrons as their relativistic mass increase even at low energies is quite high. The above difficulty is overcome in a betatron designed and developed by D.W. Kerst in 1941 at the University of Illinois. In fact, the betatron was specifically meant for acceleration of the electrons.

Basic principle : The basic principle of betatron is to accelerate electrons in a stable orbit of essentially constant radius by the application of an alternating magnetic field, called the induction field. The induction field mainly produces two effects.

(1) the increasing magnetic flux produces an electromagnetic force that accelerates the electrons along their orbits and thus increases their energy in each successive orbital revolution

(2) the varying magnetic field, acting perpendicular to the electron-orbit, simultaneously constrains the electrons move in a circualr trajectory of constant radius.

Betatron condition: It is possible to derive a condition for a stable orbit of constant radius in a betatron as follows.

As the magnetic field varies with time, the induced emf $(\in)$ is given by

$$\in = \frac{d\phi}{dt} \text{(save the sign)} \qquad \ldots (1)$$

where ϕ is the magnetic flux enclosed by the orbiting electrons,

If r be the radius of the electron orbit, the induced emf is also given by

$$\in = 2\pi r X \qquad \ldots (2)$$

where X is the induced electric field.

Therefore, the force acting on the electron of charge $\in$ is

$$F = eX = e\frac{\in}{2\pi r} = \frac{e}{2\pi r}\frac{d\phi}{dt} \quad \text{[using equations (1) and (2)]} \quad \ldots (3)$$

As the electron of mass m and velocity v is deflected by magnetic induction B along an orbit of radius r,

$$\frac{mv^2}{r} = Bev$$

Therefore, momentum, $p = mv = Ber$ ... (4)

$$\therefore \quad F = \frac{dp}{dt} = \frac{d}{dt}(Ber) = er\frac{dB}{dt}, \text{ using equation (4)} \qquad \ldots (5)$$

Comparing equations (3) and (5), $\boxed{\frac{d\phi}{dt} = 2\pi r^2 \frac{dB}{dt}}$ ... (6)

This equation is known as the betatron condition,

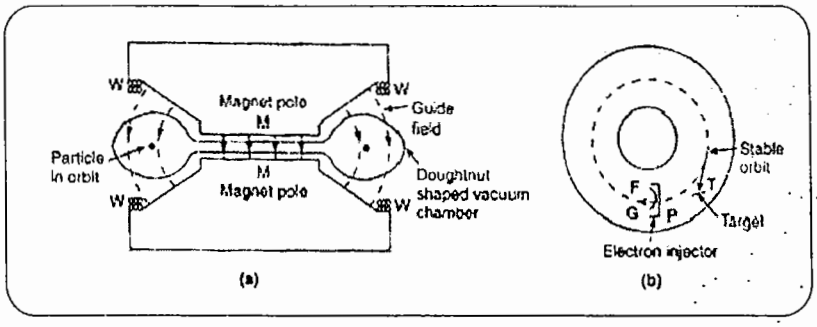
Integrating equation (6), $\phi = 2\pi r^2 B$ ... (7)

For uniform magnetic induction, however, the flux would be $\pi r^2 B$.

Thus for a stable orbit of constant radius, the pole pieces of the magnet are to be so designed that the flux must change at twice the rate at which it would change if the magnetic induction were uniform throughout the area enclosed by the orbit.

The above condition is achieved by designing pole pieces from laminated iron core in which the varying magnetic field is produced by an a.c. supply of about 60 Hz. Then the flux density at the centre of the orbit is larger than that at the edges. This condition is valid for both relativistic and non-relativistic energies of the electron.

Construction: The betatron (see figure) consists essentially of
(i) a highly evacuated doughnut-shaped glass chamber containing the source of electrons;
(ii) a specially shaped powerful electromagnet *MM* between the pole pieces of which the doughnut is mounted-the pole pieces being so designed that they have a strong field at the centre with gradient towards the edges;
(iii) the windings *WW* through which passes an alternating current to energies the



electromagnet,
(iv) the filament *F* which is the source of electrons produced thermoinically. The thermoelectrons are given an initial energy of 50 kV by applying a high potential between *F* and the plate *P*, *G* being the grid to focus the electrons. The electrons are injected into the doughnut at a time when the magnetic flux produced by the electromagnetic rises from zero in the first quarter cycle, *T* is the target which is impinged by the stream of accelerated electrons.

When the magnetic field increases from zero to the final positive maximum, the electrons complete enough number of revolutions (thousands) in the fixed stable orbit gaining higher and higher energies. When the desired energy is achieved, the radius of the orbit is suddenly increased by increasing the flux ϕ, without changing the field, as given by the relation $r = (\phi / 2\pi B)^{1/2}$, and the electrons are deflected onto the target *T*. The above is ensured by the discharge of a condenser through the primary of the electromagnet that increases the flux without changing the flux density.

Energy computation: The momentum *p* imparted to an electron of charge *e* by a field *B* is given by

$$\frac{mv^2}{r} = Bev$$

Or, $$p = mv = Ber$$

Therefore, the energy (relativistically) is : $E = pc = Berc$
where *B* is the magnetic field at the end of the acceleration time.

In a betatron, the acceleration time is $\frac{1}{4}T$, where *T* is the time-period $\left(=\frac{2\pi}{\omega}\right)$, ω being the angular frequency.

$$\therefore \quad \frac{T}{4} = \frac{1}{4} \times \frac{2\pi}{\omega} = \frac{\pi}{2\omega} \qquad \ldots (8)$$

Therefore, distance travelled by the electron in time $\frac{T}{4}$ is $d = \frac{\pi c}{2\omega}$, since the velocity of electron approaches the velocity of light *c* after being accelerated.

Therefore, Number of revolutions executed per sec by the electron is

$$n = \frac{d}{2\pi r} = \frac{\pi c}{2\omega} \times \frac{1}{2\pi r} = \frac{c}{4\omega r} \qquad \ldots (9)$$

where r is the radius of the stable orbit.

When the flux changes as a function of time, the energy gained per sec by the electron is

$$e\frac{d\phi}{dt} = e\frac{d}{dt}(\phi_0 \sin \omega t) \qquad [\because \phi = \phi_0 \sin \omega t]$$
$$= e\omega\phi_0 \cos \omega t$$

Since the acceleration occurs in time $T/4$, the average energy gained in each revolution is

$$\int_0^{T/4} e\omega\phi_0 \cos \omega t dt \Big/ \int_0^{T/4} dt = e\omega\phi_0 \int_0^{T/4} \cos \omega t dt \Big/ \int_0^{T/4} dt = \frac{2e\omega\phi_0}{\pi}$$

[since, $T/4 = \pi/2\omega$ by equation (8)]

Therefore, the total energy imparted to the electron in n revolution is

E_T = mean energy per rev. × number of revolutions

$$= \frac{2e\omega\phi_0}{\pi} \times \frac{c}{4\omega r} = \frac{ec\phi_0}{2\pi r}, \text{ using equation (9)}$$

Difference between Cyclotron and Betatron :

	CYCLOTRON		BETATRON
1.	Fixed-frequency cyclotron employs no alternating magnetic field; the magnetic field is constant.	1.	Betatron, however, employs an oscillating field which is called the induction field.
2.	Particles are accelerated in helical orbits and so orbital radius goes on increasing and equals the radius of the dees in the limit.	2.	The acceleration of the particles in a betatron takes place in a circular orbit which is essentially of constant radius.
3.	A fixed-frequency cyclotron is unsuitable for accelerating electrons	3.	A betatron is specifically meant for accelerating electrons.
4.	Cyclotron suffers from the problem of orbital stability.	4.	It also suffers from the problem of orbital stability.
5.	This cannot be used as an X-ray generator.	5.	This is a very highly efficient X-ray generator.

SOLVED PROBLEMS

1. Four partices-electron, proton, He^+ and Li^+ are projected in a circular orbit of same radius perpendicular to a given magnetic field. Then the velocity will be smallest for
(a) electron (b) He^+ (c) proton (d) Li^+

Soln. We know that radius of the circle

$$r_L = \frac{mv}{qB}$$

Since, q, B and r are fixed.

$$\therefore \quad v \propto \frac{1}{m}$$

Therefore, the velocity of He^+ will be smallest.
Correct option is (c)

2. Deuterons in a cycltron describe a circle of radius 0.32 m just before emerging from the dees. The ac voltage applied to the dees is 2×10^4 volt at 10 MHz. Find (i) the velocity of deutrons, (ii) the magnetic field and (iii) the energy of deutrons in MeV:

Solu. (i) From the relation $v = 2\pi rn$, the velocity is given by

$$v = 2\times3.1416\times0.32\times10\times10^6 = 2.01\times10^7 \text{ m/s}$$

(ii) From the relation $Bqv = mv^2/r$, we obtain the magnetic field, $B = (mv/qr)$

Here, $v = 2.01\times10^7$ m/s, m = mass of deutron = $2\times1.67\times10^{-27}$ kg, $r = 0.32$ m, $q = 1.60\times10^{-19}$ C.
Substituting these values in the expression for B, we obtain

$$\therefore \quad B = \frac{2\times1.67\times10^{-27}\times2.01\times10^7}{1.60\times10^{-19}\times0.32} = 1.31 \text{ Tesla}$$

(iii) Energy of deuterons $= mv^2/2$

$$= 2\times1.67\times10^{-27}\times(2.01\times10^7)^2/2$$

$$= 1.67\times(2.01)^2\times10^{-13} \text{ J}$$

$$= \frac{1.67\times(2.01)^2\times10^{-13}}{1.602\times10^{-13}} \text{ MeV} = \mathbf{4.22\ MeV}$$

3. A cyclotron has a magnetic field of 1.5 Wb/m^2. The extraction radius is 0.5 m. Calculate the frequency of the rf-oscillator necessary for accelerating deutrons and the energy of the extracted beam.

Soln. Frequency, $f = \frac{Bq}{2\pi m} = \frac{1.5\times1.6\times10^{-19}}{2\times3.14\times2\times1.67\times10^{-27}}$

$$= 11.44\times10^6 \text{ Hz} = \mathbf{11.44\ MHz}$$

Kinetic energy of deuterons, $W_k = \frac{B^2q^2r^2}{2m}$. Substituting the given values in the expression for W_k, we get

$$\therefore \quad W_k = \frac{(1.5)^2\times(1.6\times10^{-19})^2\times(0.5)^2}{2\times2\times1.67\times10^{-27}} = 2.155\times10^{-12} \text{ J}$$

$$= \frac{2.155\times10^{-12}}{1.6\times10^{-13}} \text{ MeV} = \mathbf{13.46\ MeV}$$

4. What is the energy to which protons can be accelerated in a cyclotron with a dee (diameter of 2 m) and a magnetic field of flux density 0.72 Wb/m^2. Mass of proton = 1.673×10^{-27} kg

Soln. From the relation, $\frac{mv^2}{r} = Bqv$, we obtain the expression for the velocity

$$v = \frac{Bqr}{m}$$

Therefore, energy, $W_k = \frac{1}{2}mv^2 = \frac{1}{2m}(Bqr)^2$

$$\therefore \quad W_k = \frac{\left(0.72\times1.6\times10^{-19}\times1\right)^2}{2\times1.673\times10^{-27}} = 3.966\times10^{-12}\,\text{J}$$

$$= \frac{3.966\times10^{-12}}{1.6\times10^{-13}}\,\text{MeV} = \mathbf{24.78\ MeV}$$

5. An alternating potential difference of 50 kV maximum value is applied to the dees of radius 40 cm of a cyclotron by an oscillator. A deuteron of mass 2 a.m.u. acquires energy of 4 MeV in the cyclotron. Calculate (i) magnetic field strength, (ii) frequency of the oscillator and (iii) number of revolutions which the deuteron has to make inside the cyclotron to gain the energy. 1 a.m.u. $=1.6\times10^{-27}$ kg, $q = 1.6\times10^{-19}$ C

Soln. Kinetic energy of the emerging beam of particles of mass m, charge q in a cyclotron is given by

$$W_k = \frac{B^2q^2r^2}{2m}$$

where, B = the magnetic field and r = the radius of a dee.

(i) $\therefore \quad B^2 = \frac{2W_km}{q^2r^2} = \frac{2\times\left(4\times1.6\times10^{-13}\right)\times\left(2\times1.6\times10^{-27}\right)}{\left(1.6\times10^{-19}\right)^2} = 1.0 \quad \therefore \quad B = \mathbf{1.0\ Wb/m^2}$

(ii) Resonance frequency, $f = \frac{Bq}{2\pi m} = \frac{1.0\times1.6\times10^{-19}}{2\times3.14\times\left(2\times1.6\times10^{-27}\right)} = 8\times10^6\ \text{c/s} = \mathbf{8\ Mc/s}$

(iii) Let n be the number of revolutions, $n = \frac{W_k}{2qV} = \frac{4\times1.6\times10^{-13}}{2\times1.6\times10^{-19}\times50\times10^3} = \mathbf{40}$

6. In accelerators electrons e^- are made to collide with their anti-particles, positrons, e^+, at high energies to produce many different particles. In one such process, the pair annihilates produce a muon μ^- and an anti-muon μ^+. The mass of a muon $m_\mu \approx 200\,m_e$, where m_e is the electrons rest mass. What is the minimum velocity to which $e^\pm$ must be accelerated so that $\mu^\pm$ may be produced ? (You may give your answer as a friction of the speed of light c). **[JNU Ph.D. 2008]**

Soln. $e^- + e^+ \rightarrow \mu^- + \mu^+$

Here, $m_\mu = 200\,m_e$.

So, $Q = (m_{e^-} + m_{e^+} - m_{\mu^-} + m_{\mu^+})c^2$

$$= (m_e + m_e - 200\,m_e - 200\,m_e)c^2$$

$$= -398\,m_e c^2$$

$$Q = -398\times0.51\ \text{MeV} \Rightarrow \boxed{Q = -202.98\ \text{MeV}}$$

$$E = -Q\left(1+\frac{m_A}{m_B}\right)$$

Here, assuming that e^- has K.E. and e^+ is in rest.

So, $E = -Q\left(1+\frac{m_{e^-}}{m_{e^+}}\right)$

$\Rightarrow E = -2Q$

$E = -2(-202.98\ \text{MeV})$

$E = 405.96\ \text{MeV}$

To find velocity of e^+

$$\frac{m_0 c^2}{\sqrt{1-v^2/c^2}} = 405.96\ \text{MeV}$$

$$\Rightarrow \sqrt{1-\frac{v^2}{c^2}} = \frac{0.51}{405.96} \Rightarrow \sqrt{1-\frac{v^2}{c^2}} = 1.26\times10^{-3} \Rightarrow 1-\left(\frac{v}{c}\right)^2 = 1.5876\times10^{-6}$$

$$\therefore \frac{v}{c} = \sqrt{1-1.5876\times10^{-6}}$$

$$\frac{v}{c} = 0.99 \Rightarrow v = 0.99\,c.$$

Correct answer is (0.99)

leration

Chapter 5

Nuclear Reactions

Introduction:

When two nuclei come close together, a nuclear reaction can occur that results in nuclei being formed. Nuclei are positively charged and the repulsion between them keeps them beyond the range where they can interact unless they are moving very fast. In the sun and stars whose internal temperatures range up to millions of Kelvins, many nuclei present have high enough speeds for reaction to be frequent. In lab it is easy to produce nuclear reactions on a small scale, either with α-particles or with protons or heavier nuclei accelerated in various ways.

Classification of nuclear reactions:

(i) **Elastic scattering:** In elastic scattering, the same particles are scattered in different directions and there is no loss of energy. The residual nucleus is the same as the target nucleus and is left in the same state (ground state) as the latter so that it can be represented as X(x, x)X. An example is the scattering of neutrons by graphite: $^{12}C(n,n)^{12}C$. **Example:** ${}_2He^4 + {}_{79}Au^{197} \rightarrow {}_{79}Au^{197} + {}_2He^4$

(ii) **Inelastic scattering:** In inelastic scattering, the same particles are scattered in different directions with different energy and there is loss of energy due to collision. The residual nucleus which is the same as target nucleus is left in an excited state so that the process can be represented as $X(x,y)X^*$. An example is the collision of fast neutrons with U-238.

(iii) **Radiative capture:** In radiative capture, the incident particle is absorbed or captured by the target nucleus to form the excited compound nucleus which disintegrates to produce one or more γ-photons and goes down to the ground state. The process may be represented as $X(x,\gamma)Y^*$.

Example: ${}_{92}U^{235} + {}_0n^1 \longrightarrow {}_{92}U^{236}$.

(iv) **Reaction of transformation:** Here the oncoming particle is retained in the nucleus and the compound nucleus emits a different particle so that the product nucleus is different from target nucleus, e.g. $^9Be(\alpha,n)^{12}C$.

(v) **Photo-disintegration:** In photo-disintegration, a very energetic photon is absorbed by the target nucleus so that it is raised to an excited state and subsequently disintegrates. It can be represented as $X(\gamma,y)Y$. **Example:** ${}_1H^2 + \gamma \longrightarrow {}_1H^1 + {}_0n^1$.

(vi) **Stripping reactions:** In stripping reactions, one or more nucleons from the projectile are captured by the target nucleus, the remaining stripped nucleus is emitted in a different direction.

Example: ${}_{29}Cu^{63} + {}_1D^2 \longrightarrow {}_{29}Cu^{64} + {}_1H^1$

(vii) **Direct reactions:** A collision of an incident particle with the nucleus may immediately pull one of the nucleons out of the target nucleus and is called 'pick up reaction'.

Example: ${}_1H^1 + {}_6C^{13} \longrightarrow {}_1D^2 + {}_6C^{12}$

Conservations in Nuclear reactions:

(i) Conservation of mass number
(ii) Conservation of atomic number
(iii) Conservation of energy (including mass-energy)
(iv) Conservation of linear momentum
(v) Conservation of angular momentum.
(vi) Conservation of parity
(vii) Conservation of isotopic spin

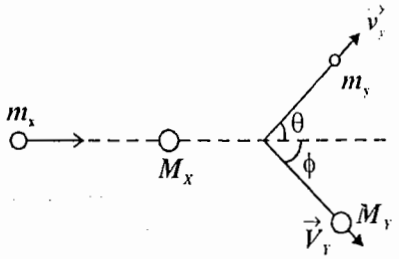
Kinematics of nuclear reactions:

Let a particle of mass m_x moving along x-axis with a velocity v_x collide elastically with a target nucleus of mass M_X, at rest. Due to the nuclear reaction, the product nucleus of mass M_Y is scattered at an angle ϕ with velocity v_Y and the product particle of m_y is emitted with velocity v_y at an angle θ.
From the law of conservation of linear momentum along x and y directions, we get

$$\left.\begin{aligned} M_Y v_Y \cos\phi + m_y v_y \cos\theta &= m_x v_x \\ -M_Y v_Y \sin\phi + m_y v_y \sin\theta &= 0 \end{aligned}\right\}$$

Or,

$$\left.\begin{aligned} M_Y v_Y \cos\phi &= m_x v_x - m_y v_y \cos\theta \\ M_Y v_Y \sin\phi &= m_y v_y \sin\theta \end{aligned}\right\}$$



Squaring the above two equations and adding, we obtain

$$M_Y^2 v_Y^2 = m_x^2 v_x^2 + m_y^2 v_y^2 - 2m_x m_y v_x v_y \cos\theta$$

Or,
$$2M_Y K_Y = 2m_x K_x + 2m_y K_y - 4\sqrt{m_x m_y K_x K_y}\cos\theta$$

since, non-relativistically, K = kinetic energy $= \frac{1}{2}mv^2$

$$\therefore \quad K_Y = \frac{m_x}{M_Y}K_x + \frac{m_y}{M_Y}K_y - \frac{2}{M_Y}\sqrt{m_x m_y K_x K_y}\cos\theta$$

But, $Q = K_Y + K_y - K_x$

$$\therefore \quad Q = K_y\left(1+\frac{m_y}{M_Y}\right) - K_x\left(1-\frac{m_x}{M_Y}\right) - \frac{2}{M_Y}\sqrt{m_x m_y K_x K_y}\cos\theta$$

This gives the Q-value of the reaction in terms of K_y, K_x and θ without involving the kinetic energy of the recoil nucleus, K_Y and the mass M_x of target nucleus and is called the standard form of Q-equation.

If $\theta = 90°$, the Q-expression simplifies to: $Q = K_y\left(1+\frac{m_y}{M_Y}\right) - K_x\left(1-\frac{m_x}{M_Y}\right)$

If Q is negative, it gives the minimum value of threshold kinetic energy to be given to the incident particle for an endoergic or endothermic reaction to proceed in the forward direction.

Q-value and threshold energy of nuclear reaction:

The law of conservation of energy and momentum imposes certain restrictions on the reactions. These restrictions are called as the kinematic restrictions and this mathematical method is known as kinematics. Consider the nuclear reaction

$$x + X \longrightarrow y + Y$$

Where x, X, y and Y are the bombarding particle, target nucleus, outgoing particle and product nucleus respectively. It is assumed that the target nucleus is in rest. Since total energy is conserved in the nuclear reaction, therefore we get,

$$\left(m_x c^2 + E_x\right) + M_X c^2 = \left(E_y + m_y c^2\right) + \left(E_Y + M_Y c^2\right)$$

E_x, E_y and E_Y are the kinetic energies of respective particles.

Now the quantity $Q = E_y + E_Y - E_x \Rightarrow Q = \left(m_x + M_X - m_y - M_Y\right)c^2$

Where Q is called the Q-value of nuclear reaction.

(i) If Q is positive, the reaction is said to be exoergic (exothermic) and
(ii) If Q is negative, the reaction is called endoergic (endothermic).

The minimum K.E. required for incident particle (x) to start the nuclear reaction is called the threshold energy $(E_x)^{th}$. The relation between Q-values and threshold energy is:

$$\boxed{E_x^{th} = -Q\frac{\left(m_y + M_Y\right)}{\left(M_Y + m_y - m_x\right)}}$$

If $E_x^{th} = 0$ for exoergic or exothermic reactions i.e. these reaction are spontaneous process.

Mechanism of nuclear reactions:

(1) Transmutation by protons:

(a) (p-α) reactions:

(i) ${}_3Li^7 + {}_1H^1 \rightarrow \left({}_4Be^8\right)^* \rightarrow {}_2He^4 + {}_2He^4$

(ii) ${}_3Li^6 + {}_1H^1 \rightarrow \left({}_4Be^7\right)^* \rightarrow {}_2He^3 + {}_2He^4$

(iii) ${}_4Be^9 + {}_1H^1 \rightarrow \left({}_5B^{10}\right)^* \rightarrow {}_3Li^6 + {}_2He^4$

(iv) ${}_5B^{11} + {}_1H^1 \rightarrow \left({}_6C^{12}\right)^* \rightarrow {}_4Be^8 + {}_2He^4$

(v) ${}_9F^{19} + {}_1H^1 \rightarrow \left({}_{10}Ne^{20}\right)^* \rightarrow {}_8O^{16} + {}_2He^4$

(b) (p–n) Reaction:

(i) ${}_5B^{11} + {}_1H^1 \rightarrow \left({}_6C^{12}\right)^* \rightarrow {}_6C^{11} + {}_0n^1$

(ii) ${}_8O^{18} + {}_1H^1 \rightarrow \left({}_9F^{19}\right)^* \rightarrow {}_9F^{18} + {}_0n^1$

(iii) ${}_{11}Na^{23} + {}_1H^1 \rightarrow \left({}_{12}Mg^{24}\right)^* \rightarrow {}_{12}Mg^{23} + {}_0n^1$

(iv) ${}_{28}Ni^{58} + {}_1H^1 \rightarrow \left({}_{29}Cu^{59}\right)^* \rightarrow {}_{29}Cu^{58} + {}_0n^1$

(v) ${}_{29}Cu^{65} + {}_1H^1 \rightarrow \left({}_{30}Zn^{66}\right)^* \rightarrow {}_{30}Zn^{65} + {}_0n^1$ (usually endoergic)

(c) (p, d) reaction:

(i) ${}_3Li^7 + {}_1H^1 \rightarrow {}_3Li^6 + {}_1H^2$

(ii) ${}_4Be^9 + {}_1H^1 \rightarrow {}_4Be^8 + {}_1H^2$

(d) Proton capture:

Compound nucleus in excited states come to ground state with γ-ray photon.

(i) ${}_3Li^7 + {}_1H^1 \rightarrow \left({}_4Be^8\right)^* \rightarrow {}_4Be^8 + \gamma$

(ii) ${}_6C^{12} + {}_1H^1 \rightarrow \left({}_7N^{13}\right)^* \rightarrow {}_7N^{13} + \gamma$

(iii) ${}_7N^{14} + {}_1H^1 \rightarrow \left({}_8O^{15}\right)^* \rightarrow {}_8O^{15} + \gamma$

(iv) ${}_9F^{19} + {}_1H^1 \rightarrow \left({}_{10}Ne^{20}\right)^* \rightarrow {}_{10}Ne^{20} + \gamma$

(v) ${}_{13}Al^{27} + {}_1H^1 \rightarrow \left({}_{14}Si^{28}\right)^* \rightarrow {}_{14}Si^{28} + \gamma$

(vi) ${}_{24}Cr^{50} + {}_1H^1 \rightarrow \left({}_{25}Mn^{51}\right)^* \rightarrow {}_{25}Mn^{51} + \gamma$

If incident proton has an energy (> 20 MeV) the compound nucleus has sufficient excitation energy to permit the expulsion of two or more nucleons.

(2) Transmutation by Neutrons:-

Neutrons have no electric charge and can penetrate positively charged nuclei without any experience of repulsive electrostatic force.

(a) ($n-\alpha$) reaction:

With slow Neutrons

(i) ${}_3Li^6 + {}_0n^1 \rightarrow \left({}_3Li^7\right)^* \rightarrow {}_1H^3 + {}_2He^4$ (ii) ${}_5B^{10} + {}_0n^1 \rightarrow \left({}_5B^{11}\right)^* \rightarrow {}_3Li^7 + {}_2He^4$

(iii) ${}_{13}Al^{27} + {}_0n^1 \rightarrow \left({}_{13}Al^{28}\right)^* \rightarrow {}_{11}Na^{24} + {}_2He^4$ (iv) ${}_7N^{14} + {}_0n^1 \rightarrow \left({}_7N^{15}\right)$

(v) ${}_5B^{11} + {}_2He^4$ (vi) ${}_3Li^7 + {}_2He^4 + {}_2He^4$

Capture of fast Neutrons - emission of α-particle are usually radioactive

(i) ${}_{11}Na^{23} + {}_0n^1 \rightarrow \left({}_{11}Na^{24}\right)^* \rightarrow {}_9F^{20} + {}_2He^4$ followed by ${}_9F^{20} \rightarrow {}_{10}Ne^{20} + \beta^-$

(ii) ${}_{13}Al^{27} + {}_0n^1 \rightarrow \left({}_{13}Al^{28}\right)^* \rightarrow {}_{11}Na^{24} + {}_2He^4$ followed by ${}_{11}Na^{24} \rightarrow {}_{12}Mg^{24} + \beta^-$

(b) (n, p) reaction: Proton in the nucleus is replaced by neutron, mass no. does not change but charge decreases by one unit.

(i) ${}_7N^{14} + {}_0n^1 \rightarrow \left({}_7N^{15}\right)^* \rightarrow {}_6C^{14} + {}_1H^1 + Q$ followed by ${}_6C^{14} \rightarrow {}_7N^{14} + \beta^- + E_{max}$

(ii) ${}_2He^3 + {}_0n^1 \rightarrow \left({}_2He^4\right)^* \rightarrow {}_1H^3 + {}_1H^1 + Q$ followed by ${}_1H^3 \rightarrow {}_2He^3 + \beta^- + E_{max}$

(iii) ${}_{17}Cl^{35} + {}_0n^1 \rightarrow \left({}_{17}Cl^{36}\right)^* \rightarrow {}_{16}S^{35} + {}_1H^1 + Q$ followed by ${}_{16}S^{35} \rightarrow {}_{17}Cl^{35} + \beta^- + E_{max}$

With fast neutrons:

(i) ${}_{13}Al^{27} + {}_0n^1 \rightarrow \left({}_{13}Al^{28}\right)^* \rightarrow {}_{12}Mg^{27} + {}_1H^1$ (ii) ${}_{16}S^{32} + {}_0n^1 \rightarrow \left({}_{16}S^{33}\right)^* \rightarrow {}_{15}P^{32} + {}_1H^1$

(iii) ${}_{29}Cu^{65} + {}_0n^1 \rightarrow \left({}_{29}Cu^{66}\right)^* \rightarrow {}_{28}Ni^{65} + {}_1H^1$ (iv) ${}_{30}Zn^{64} + {}_0n^1 \rightarrow \left({}_{30}Zn^{65}\right)^* \rightarrow {}_{29}Cu^{64} + {}_1H^1$

(c) ($n-\gamma$) Reaction:

(i) ${}_1H^1 + {}_0n^1 \rightarrow \left({}_1H^2\right)^* \rightarrow {}_1H^2 + \gamma$ (ii) ${}_1H^2 + {}_0n^1 \rightarrow \left({}_1H^3\right)^* \rightarrow {}_1H^3 + \gamma$

Product Nucleus is Radioactive:

(i) ${}_{45}Rh^{103} + {}_0n^1 \rightarrow \left({}_{45}Rh^{104}\right)^* \rightarrow {}_{45}Rh^{104} + \gamma$ (ii) ${}_{49}In^{115} + {}_0n^1 \rightarrow \left({}_{49}In^{116}\right)^* \rightarrow {}_{49}In^{116} + \gamma$

(iii) ${}_{92}U^{238} + {}_0n^1 \rightarrow \left({}_{92}U^{239}\right)^* \rightarrow {}_{92}U^{239} + \gamma$ (iv) ${}_{29}Cu^{65} + {}_0n^1 \rightarrow \left({}_{29}Cu^{66}\right)^* \rightarrow {}_{29}Cu^{66} + \gamma$

(v) ${}_{79}Au^{65} + {}_0n^1 \rightarrow \left({}_{79}Au^{198}\right)^* \rightarrow {}_{79}Au^{198} + \gamma$

(d) ($n-d$) and ($n-t$) reaction: Bombardment of 90 MeV neutrons: ${}_7N^{14} + {}_0n^1 \rightarrow {}_6C^{12} + {}_1H^3$

(e) ($n-2n$) reaction: one neutron captured by nucleus and 2 neutrons are emitted. $Q < 0$ fast neutron are needed. Most cases residue nucleus unstable-followed by positron emission.

(i) ${}_6C^{11} + {}_0n^1 \rightarrow \left({}_6C^{13}\right)^* \rightarrow {}_6C^{11} + 2\,{}_0n^1$ (ii) ${}_{19}K^{39} + {}_0n^1 \rightarrow \left({}_{19}K^{40}\right)^* \rightarrow {}_{19}K^{38} + 2\,{}_0n^1$

(iii) ${}_{51}Sb^{121} + {}_0n^1 \rightarrow \left({}_{51}Sb^{122}\right)^* \rightarrow {}_{51}Sb^{120} + 2\ {}_0n^1$

(f) **Neutron - Three or more particles:** Incident Neutron ~ 30 MeV sufficient energy to overcome coulomb Barrier, 3 neutrons or even 2 neutron and a proton are ejected from compound nucleus. Neutron (~ 100 MeV) Nuclei with moderate mass number undergo spallation and those of high mass number eg. *Bi* and *Pb* suffer fission probably accompanied by spallative.

(3) **Transmutation by Deutron:** High energy Deutron.

(a) (d–α) reaction

(i) ${}_3Li^6 + {}_1H^2 \rightarrow \left({}_4Be^8\right)^* \rightarrow {}_2\ He^4 + {}_2\ He^4$

(ii) ${}_8O^{16} + {}_1H^2 \rightarrow \left({}_9F^{18}\right)^* \rightarrow {}_7\ N^{14} + {}_2\ He^4$,

(iii) ${}_{10}Ne^{20} + {}_1H^2 \rightarrow \left({}_{11}Na^{22}\right)^* \rightarrow {}_9\ F^{18} + {}_2\ He^4$

(iv) ${}_{12}Mg^{26} + {}_1H^2 \rightarrow \left({}_{13}Al^{28}\right)^* \rightarrow {}_{11}\ Na^{24} + {}_2\ He^4$,

(v) ${}_{13}Al^{27} + {}_1H^2 \rightarrow \left({}_{14}Si^{29}\right)^* \rightarrow {}_{12}\ Mg^{25} + {}_2\ He^4$

(b) (d–p) Reaction: Isotope creation

(i) ${}_6C^{12} + {}_1H^2 \rightarrow \left({}_7N^{14}\right)^* \rightarrow {}_6\ C^{13} + {}_1\ H^1$

(ii) ${}_3Li^7 + {}_1H^2 \rightarrow \left({}_4Be^9\right)^* \rightarrow {}_3\ Li^8 + {}_1\ H^1$

(iii) ${}_{11}Na^{23} + {}_1H^2 \rightarrow \left({}_{12}Mg^{25}\right)^* \rightarrow {}_{11}\ Na^{24} + {}_1\ H^1$

(iv) ${}_{15}P^{31} + {}_1H^2 \rightarrow \left({}_{16}S^{33}\right)^* \rightarrow {}_{15}\ P^{32} + {}_1\ H^1$

(v) ${}_{48}Cd^{114} + {}_1H^2 \rightarrow \left({}_{49}In^{116}\right)^* \rightarrow {}_{48}\ Cd^{115} + {}_1\ H^1$

(vi) ${}_{83}Bi^{209} + {}_1H^2 \rightarrow \left({}_{84}Po^{211}\right)^* \rightarrow {}_{83}\ Bi^{210} + {}_1\ H^1$

(c) (d–n) Reaction:

(i) ${}_3Li^7 + {}_1H^2 \rightarrow \left({}_4Be^9\right)^* \rightarrow {}_4\ Be^8 + {}_0\ n^1$

(ii) ${}_4Be^9 + {}_1H^2 \rightarrow \left({}_5B^{11}\right)^* \rightarrow {}_5\ B^{10} + {}_0\ n^1$

(iii) ${}_6C^{12} + {}_1H^2 \rightarrow \left({}_7N^{14}\right)^* \rightarrow {}_7\ N^{13} + {}_0\ n^1$

When two Deutrons Interact both the (d, n) and (d, p) reactions have been obtained.

${}_1H^2 + {}_1H^2 \rightarrow {}_2\ He^4 \rightarrow {}_1H^3 + {}_1H^1 + 4.02\ \text{MeV} \rightarrow {}_2He^3 + {}_0n^1 + 3.25\ \text{MeV}$

Nuclear fission and fusion :

Fission and fusion are two processes that alter the nucleus of an atom. Nuclear fission provides the energy in nuclear power plants and fusion is the source of the sun's energy. The use of fission in power plant can help conserve fossil fuels. Without the energy produced by the fusion of hydrogen in the sun, the earth would quickly change into a cold planet that could not support life as we know it.

Nuclear transformation always obey two fundamental conservation laws

(i) Mass number is conserved

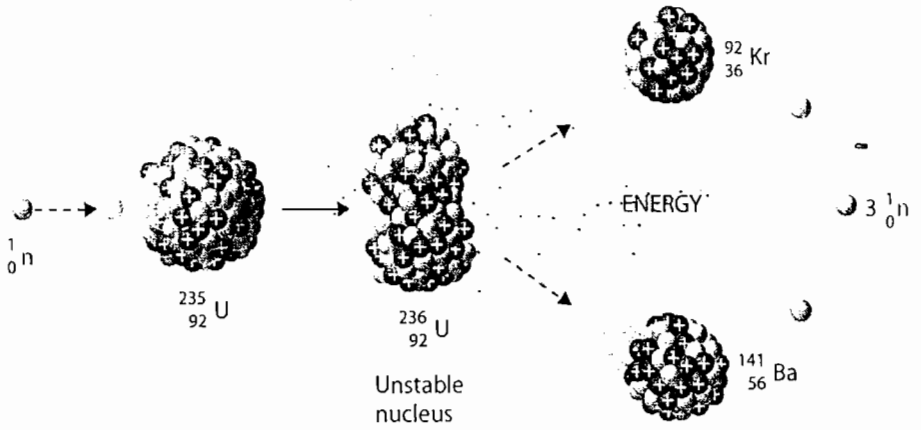
(ii) Electric charge is conserved.

Energy and mass are not conserved but can be interconverted according to Einstein's equation.

$$\boxed{E = mc^2}$$

Nuclear fission:

The process of fission occurs when a nucleus splits into smaller pieces. Fission can be induced by a nucleus capturing slow moving neutrons, which result in the nucleus becoming very unstable.



The following equation represent fission reactions

$$^{235}_{92}U + {}^{1}_{0}n \longrightarrow {}^{141}_{56}Ba + {}^{92}_{36}Kr + 3{}^{1}_{0}n$$

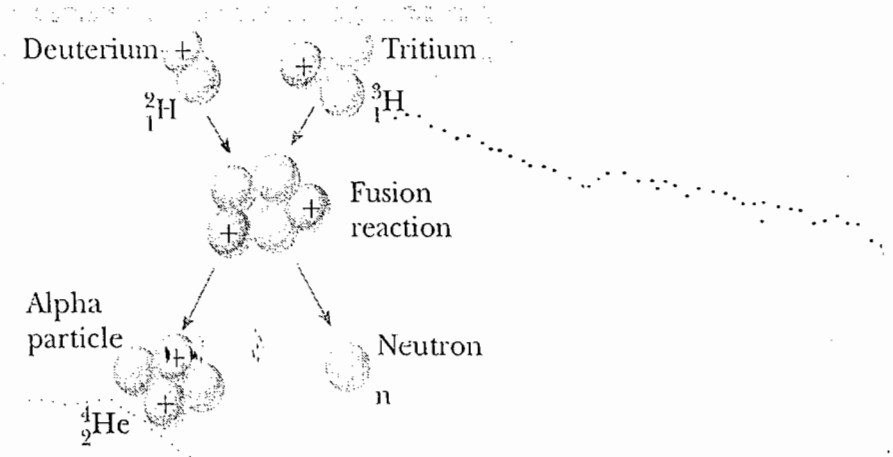
$$^{235}_{92}U + {}^{1}_{0}n \longrightarrow {}^{137}_{52}Te + {}^{97}_{40}Zr + 2{}^{1}_{0}n$$

$$^{235}_{92}U + {}^{1}_{0}n \longrightarrow {}^{152}_{66}Nd + {}^{81}_{32}Ge + 3{}^{1}_{0}n$$

$$^{235}_{92}U + {}^{1}_{0}n \longrightarrow {}^{131}_{50}Sn + {}^{103}_{42}Mo + 2{}^{1}_{0}n_{0}$$

Fusion :

Fusion occurs when two nuclei joint together to form a large nucleus. Fusion is brought about by bringing together two or more small nuclei under conditions of tremendous pressure and heat.



The following equation represent fusion reactions

$$^{2}_{1}H + {}^{2}_{1}H \longrightarrow {}^{3}_{1}H + {}^{1}_{1}H$$

$$^{2}_{1}H + {}^{3}_{1}H \longrightarrow {}^{4}_{2}He + {}^{1}_{0}n$$

$$^{3}_{2}He + {}^{3}_{2}He \longrightarrow {}^{4}_{2}He + 2{}^{1}_{1}H$$

Condition of fusion reaction of heat :

Thermal kinetic energy = Coulomb's repulsion

$$\Rightarrow \quad \boxed{\frac{3}{2}kT = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}}}$$

Cross-section of nuclear reactions (classical):

One of the most important parameters in nuclear reactions is the reaction cross-section, symbolised by σ. It is quantitative measure of the probability of occurrence of a nuclear reaction.

Let a parallel beam of N monoenergetic particles be incident per unit time normally on a target foil of surface area A and thickness Δx, having n nuclei per unit volume. Now, the number ΔN of nuclei in the foil undergoing nuclear reaction will be proportional to (i) the intensity of the beam and (ii) the number of target nuclei contained in the foil.

But the intensity I of the beam $= N/A$ and the number of nuclei in the foil $= nA.\Delta x$

Therefore, number of nuclei transmuted is $\Delta N \propto \frac{N}{A} nA.\Delta x$

$\Delta N = \sigma N n.\Delta x = \sigma N n_1$ where σ = constant.

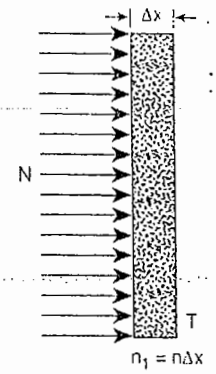


Figure: Nuclear reaction cross-section: bombardment of target foil.

where, $n_1 = n.\Delta x$, the number of nuclei per unit area of the target foil used.

The constant σ is called the nuclear reaction cross-section.

Therefore, nuclear reaction cross-section, $\boxed{\sigma = \frac{\Delta N}{N n_1}}$

- The probability of the incident particle to interact with the target atom will be equal to

$$P = \frac{\text{number of interacting particle}}{\text{number of incident particle}}$$

If dN is number of interacting particle and N is number of incident particles, then

$$\boxed{P = \frac{dN}{N} = \sigma\, n\, dx} \Rightarrow \int_{N_0}^{N} \frac{dN}{N} = \int_{x=x}^{x=0} \sigma\, n\, dx \Rightarrow \ell n\left[\frac{N}{N_0}\right] = -\sigma\, n\,(+x) \Rightarrow \frac{N}{N_0} = e^{-\sigma\, nx}$$

$$\boxed{N = N_0\, e^{-\sigma\, n\, x}}$$

Neutron - Proton scattering at low energies

- Nucleus is a bound system means attractive force exists between neutron and proton.
  In scattering of free neutrons by protons a parallel beam of Neutron is allowed to impinge upon a target containing hydrogen atoms and number of Neutrons dejected through various angles is determined as a function of Neutron energy.

- Since neutron have no charge, they are unaffected by the electrostatic field and their scattering will directly reflect the operation of nuclear force.

Two kinds of reactions can be involved in Neutron-proton interaction

(i) Scattering

(ii) Radiative Capture: It has low probability for high energy neutron as the cross-section for the competing radiative capture reaction decreases with $\frac{1}{2} \rightarrow$ neutron velocity

In practice protons are bound in nucleus the chemical binding energy of the proton in a molecule is about 0.1eV.

Thus for neutron energy > 1 eV proton can be assumed free. This sets lower limit the neutron energy.

If the neutron energy is less than 10 MeV, only the *S*-wave overlaps with the Nuclear potential and is scattered.

In the centre of mass system, the Schrodinger equation for the two body (n–p) system is

$$\nabla^2\Psi + \frac{M}{\hbar^2}\left[E - V(r)\right]\Psi = 0$$

Where M = proton or Neutron mass = 2 × reduced mass of the system.

E = Incident K.E. in cm system = $\frac{1}{2}$(Incident K.E. in L-coordinate) and $V(r)$ = Inter Nucleon potential energy.

At large distances from the centre of scattering the solution of this equation is expected to be of the form

$$\psi = \phi_{inc}(\bar{r}) + \phi_{sc}(\bar{r})$$

$\phi_{inc} = A\,e^{i(\bar{k}_0\cdot\bar{r})}$ incident wave function

ϕ_{sc} = scattering wave function

$$= A\,e^{i(\bar{k}_0\cdot\bar{r})} + Af(\theta)\frac{e^{i(\bar{k}\cdot\bar{r})}}{r}$$

where, $f(\theta)$ is the scattering amplitude in a direction θ and the differential crosssection is given by

$$\frac{d\sigma}{d\Omega} = |f(\theta)|^2$$

If we assume that the potential is spherically symmetric and scattering is ellastic scattering, then amplitude is given by

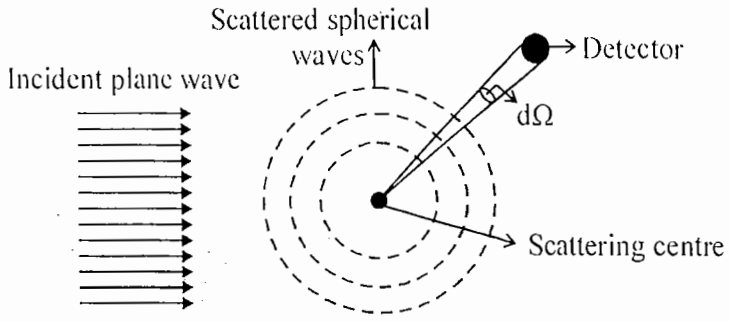
$$\boxed{f(\theta) = \frac{1}{k}\sum_{\ell=0}^{\infty}(2\ell+1)e^{i\delta_\ell}\sin\delta_\ell P_\ell(\cos\theta)}$$

Total crosssection is given by

$$\sigma = \int |f(\theta)|^2\, d\Omega$$

$$\Rightarrow \quad \sigma = 2\pi\int |f(\theta)|^2 \sin\theta\, d\theta$$

$$\Rightarrow \quad \boxed{\sigma = \frac{4\pi}{k^2}\sum(2\ell+1)\sin^2\delta_\ell}$$



Inelastic scattering:

It $\eta_\ell(k)$ is the loss factor and $0<\eta_\ell(k)\le 1$ for enelastic scattering, scattering amplitude

$$f(\theta)=\frac{1}{2k}\sum_{\ell=0}^{\infty}(2\ell+1)\left[\eta_\ell\sin 2\delta_\ell+i\left(1-\eta_\ell\cos\delta_\ell\right)\right]p_\ell(\cos\theta)$$

Total elastic scattering cross section is given by

$$\boxed{\sigma_{el}=\frac{\pi}{k^2}\sum_{\ell=0}^{\infty}(2\ell+1)\left(1+\eta_\ell^2-2\eta_\ell\cos 2\delta_\ell\right)}$$

Total inelastic scattering cross section is given by

$$\boxed{\sigma_{inel}=\frac{\pi}{k^2}\sum_{\ell=0}^{\infty}(2\ell+1)\left(1-\eta_\ell^2(k)\right)}$$

And the total cross-section is given by

$$\boxed{\sigma_{tot}=\frac{2\pi}{k^2}\sum(2\ell+1)\left(1-\eta_\ell\cos\left(2\delta_\ell\right)\right)}$$

* $\eta_\ell=0$ pure inelastic scattering.

$\eta_\ell=1$ for pure elastic scattering.

- **Scattering length:**
- For neutrons of very low energy scattered by free protons $\lambda\!\!\!^{-}$ is very large and hence k is very small, we have $f(\theta)=\frac{e^{i\delta_0}}{k}\sin\delta_0$ and scattering cross-section $\sigma_{sc}=4\pi\lambda\!\!\!^{-2}\sin^2\delta_0$.

  As $k\to 0$, δ_0 must approach zero or $f(\theta)$ would become infinite.
- For low energy neutrons $f(\theta)$ can be written as $f(\theta)=\lim_{\delta_0\to 0}\frac{e^{i\delta_0}\sin\delta_0}{k}=\frac{\delta_0}{k}=-a$

  where quantity $+a$ is called scattering length.
- The zero energy scattering cross-section is $\sigma_0=4\pi a^2$ (Identical to impenetrable sphere of radius a).

Deutron:

Deutron is the only two-nucleon bound system made up of a proton and a neutron. The two other possible two nucleon system the Diproton and Dineutron do not exist as bound system.

- **Experimentally determined properties of deutron**

(i) The *B.E.* of Deutron is very small. Its value is 2.225 ± 0.003 MeV. This is very less as compared to the stable nuclei. i.e, 8 MeV, hence loosely bound.

(ii) The Angular momentum quantum number often called the nuclear spin of ground state of the Deutron is 1. It suggest that spins are parallel (triplet state) and the orbital angular momentum of the Deutron about their common center of mass is zero. The ground state is ^3S state.

(iii) The sum of the magnetic dipole moments of the proton (2.79275 μ_N) and Neutron (–1.91315 μ_N), do not exactly equal to magnetic moment of Deutron (0.85735 μ_N)

(iv) Deutron has small but positive quadrupole moment suggesting that it deviates from spherical shape and has the probability of finding in the next higher state i.e., ^3D state also. Hence, the state is a mixture of ^3S and ^3D state.

(v) *n-p* Combination, neutron is uncharged hence the force is not electrostatic, mass is less hence no gravitational , must be of nuclear origin and assumed to be central and attractive. This assumption somewhat disagrees with the experiment (central).

- Consider the example $\mu_p = 2.79281 \pm 0.00004$ nm and $\mu_n = -1.913148 \pm 0.000066$ nm

  The fact that $\mu_d + \mu_n - \mu_d \neq 0$ although small. This suggests that the Deutron may not be fully described by the spherically symmetric $^3S_1$ state.
- Moreover, nuclear force if due to the exchange of Mesons, the magnetic moments of the Nucleons where in the free state may not be same as when in the nucleus. Correction to magnetic moment due to Mesonic current.
- *D*-state contribution can be considered by Non-Central Tensor force.
  By Assuming Central potential.

  Schrodinger equation $-\frac{\hbar^2}{2\mu}\nabla^2\psi(r) + V(r)\psi(r) = E\psi(r)$

  where reduced mass $\mu = \frac{m_p m_n}{m_p + m_n} = M/2.$

  Now equation can be written as $\frac{\hbar^2}{M}\frac{d^2u(r)}{dr^2} + V(r)u(r) = Eu(r)$

  where $\psi(r) = \frac{u(r)}{r}$ and $E = -W = -2.226$ MeV = binding energy of Deutron
- **Various types of potential**

  Square well potential $V(r) = -V_0 \quad r \leq r_0$

  $= 0 \quad r > r_0$

Exponential $V(r) = -V_0 e^{-r/r_0}$, Gaussion, $V(r) = -V_0 e^{-r^2/r_0^2}$

Yukawa $V(r) = \dfrac{-V_0 e^{-r/r_0}}{r/r_0}$

- **Ground state of deutron** $\left({}_1D^2\right)$

Deutron is a single two nucleon (one p and one n) bound system which is found in nature. The important experimental determinal properties about deutron are given below:

(i) The $B.E.$ of deutron/nucleon is very small compare to other nuclei i.e. it is a weakly system.

(ii) The ground state spin of deutron $I_d = 1$ (iii) The ground state parity of deutron = even (positive)

(iv) The quadrupole momento of deutron $(Q_d) \neq 0$

(v) The magnetic moment of deutron is slightly different from the sum of intrinsic magnetic moments of neutron + proton i.e. $(\mu_n + \mu_p) - \mu_d \to 0$

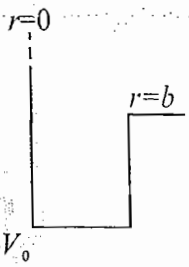
These factor represents that the ground state of Deutron is a mixture of ${}^3S_{(L=0)}$ and ${}^3D_{(L=2)}$ states in which ${}^3S$ contribution is 96% and ${}^3D$ contribution is only 4%.

This indicate that nuclear force are non-central as $L \neq$ constant and they are spin dependent. In deutron both p and n have spin parallel to each other.

- **Schrodinger wave equation for Deutron and its solution for central force ($L = 0$):**

As deutron is a bound system, let us consider the case of rectangular potential well represented as

$$\begin{bmatrix} V = -V_0 & , & 0 < r < b \\ = 0 & , & \text{elsewhere} \end{bmatrix}$$



The schrodinger equation is $\boxed{\nabla^2\psi + \frac{2\mu}{\hbar^2}(E-V)\psi = 0}$

$$\psi = \psi(r,\theta,\phi) = \sum_{\ell} R_\ell(r)\, Y_{\ell m}(\theta,\phi) = \sum \frac{u_\ell(r)}{r} Y_{\ell m}(\theta,\phi)$$

where reduce mass $\mu = \dfrac{m_n m_p}{m_n + m_p} \Rightarrow \mu \sim M/2 \left(m_n \sim m_p = M\right)$

For $\ell = 0$. We get, $\boxed{k_1 \cot k_1 b = -\alpha}$

or $$\cot k_1 b = -\frac{\alpha}{k_1} = -\frac{\sqrt{\frac{ME_d}{\hbar^2}}}{\sqrt{\frac{M}{\hbar^2}(V_0 - E_d)}} = -\sqrt{\frac{E_d}{V_0 - E_d}}$$

as E_d is very small, then an approximate solution of equation is obtained as $\cot\sqrt{\frac{M}{\hbar^2}(V_0 - E_d)}\, b \to 0$

$$\Rightarrow \quad \cot\sqrt{\frac{M}{\hbar^2}V_0}.b = \cot \pi/2, \cot 3\pi/2 .....$$

A minimum value of V_0 can be obtained by setting $b\sqrt{\frac{M}{\hbar^2}V_{0m}} = \frac{\pi}{2}$

$$\Rightarrow \qquad V_{0m} = \frac{\pi^2\hbar^2}{4Mb^2} \Rightarrow \boxed{V_{0m}b^2 = \text{constant}}$$

$V_0 \sim 25$ MeV is obtained by putting b equal to 2×10^{-15} m.

The exact solution of equation (9) is obtained by graphical method in which $V_{0m} \sim 38$ MeV.

SOLVED PROBLEMS

1. Calculate the energy generated in MeV when 0.1 kg of ^7Li is converted to ^4He by proton bombardment. Given: masses of ^7Li, ^4He and ^1H in a.m.u. are 7.0183, 4.0040 and 1.0081 respectively.

Soln. $^7_3Li + {}^1_1H = 2\,{}^4_2He + Q$

Masses of the reactants	= (7.0183 + 1.0081) a.m.u.	= 8.0264 a.m.u.
Masses of the products	= (2×4.0040) a.m.u.	= 8.0080 a.m.u.
	Therefore, Difference	= 0.0184 a.m.u.

Energy liberated in reaction = 0.0184 uc² = 0.0184 ×931.5 = 17.14 MeV

7g of Li (Lithium) contains 6.022×10^{23} atoms

So, 0.1 kg of Lithium contains $= \frac{6.022\times10^{23}}{7}\times100$ atoms

So, total energy liberated $= \frac{6.022\times10^{25}}{7}\times17.14 = 14.74\times10^{25}\ MeV$

2. Calculate the binding energy (in MeV) of ^4He from the following data: Mass of ^4He = 4.003875 a.m.u; mass of ^1H = 1.008145 a.m.u. and mass of a neutron = 1.008986 a.m.u.

Soln. A ^4He-nucleus consists of 2 protons and 2 neutrons.

Mass of (2 protons + 2 neutrons)	= 2(1.008145 + 1.008986) a.m.u.
	= 4.034262 a.m.u.
Mass of ^4He-nucleus	= 4.003875 a.m.u.
Therefore, Mass difference	= 0.030387 a.m.u.

Binding energy = 0.030387 a.m.u. × c^2 = (931×0.030387) a.m.u. = 28.29 MeV

3. Calculate the threshold energy for the nuclear reaction $^{14}N(n,\alpha)\,^{11}B$ in MeV

Soln. Masses of reactants = (14. 007550 + 1.008987) a.m.u. = 15.016537 a.m.u.
Masses of products = (4.003879 + 11.012811) a.m.u. = 15.016690 a.m.u.
Therefore,

$$Q = (15.016537 - 15.016690)\,\text{a.m.u.} = -0.000153\,\text{a.m.u.} = -0.000153\times931\ \text{MeV} = -0.14\ \text{MeV}$$

$$\Rightarrow E_{th} = -Q\left[1+\frac{M(n)}{M(N)}\right] = 0.14\left(1+\frac{1.008987}{14.007550}\right)\text{MeV} = 0.14\left(1+\frac{1}{14}\right)\text{MeV} = 0.15\ \text{MeV}$$

4. Find the amount of energy in joule released during the process in which 0.001 kg of radium is converted into lead (masses: ^{226}Ra = 226.0955 a.m.u., ^{206}Pb = 206.0386 a.m.u. and α-particle = 4.003 a.m.u.)

Soln. In the conversion of 1 atom of ^{226}Ra into 1 atom of ^{206}Pb, 5α-particle are emitted in all.

Initial mass of ^{226}Ra = 226.0955 a.m.u.
Final mass of ^{206}Pb = 206.0386 a.m.u.

Therefore, difference in masses = 20.0569 a.m.u.
Mass of 5α-particle = 20.0150 a.m.u.

Therefore, Mass converted into energy = 0.0419 a.m.u.

This is equivalent to an energy (0.0419 ×931) a.m.u. or 39 MeV (for 1 atom)

Now, 0.001 kg (= 1 g) of radium contains $\frac{6.023\times10^{23}}{226}$ Ra-atoms

Therefore, Total energy released, $E=\frac{6.023\times10^{23}\times39}{226}\text{MeV}=\frac{6.023\times10^{23}\times39\times1.6\times10^{-13}}{226}\text{J}$

$=16.63\times10^{9}$ J

5. The Q-values (in MeV) of the following three reactions

$^{2}_{1}H+^{2}_{1}H\rightarrow{}^{3}_{1}H+{}^{1}_{1}H$; $^{2}_{1}H+^{2}_{1}H\rightarrow{}^{3}_{2}H+{}^{1}_{0}n$ and $^{3}_{1}H\rightarrow{}^{2}_{3}He+\beta^{-}$

are 4.031, 3.265 and 0.0185 a.m.u. respectively. Calculate the mass difference between the neutron and the Hydrogen atom from these data.

Soln. We have: $Q_1=4.031\,\text{MeV}=\left(\text{mass of }2^{2}_{1}H-\text{mass of }{}^{3}_{1}H-\text{mass of }{}^{1}_{1}H\right)\times c^2$

$Q_2=3.265\ \text{MeV}=\left(\text{mass of }2^{2}_{1}H-\text{mass of }{}^{3}_{2}H-\text{mass of }{}^{1}_{0}n\right)\times c^2$

$Q_3=0.0185\,\text{MeV}=\left(\text{mass of }{}^{3}_{1}H-\text{mass of }{}^{3}_{2}He\right)\times c^2$

The mass of β^- is too small to be taken into account and has been neglected

$\Rightarrow\quad Q_1-Q_2+Q_3=\left(\text{mass of }{}^{1}_{0}n-\text{mass of }{}^{1}_{1}H\right)\times c^2$

Therefore, Mass difference between neutron and H-atom is thus

$Q_1-Q_2+Q_3=(4.031-3.265+0.0185)\times931.5\,\text{MeV}=0.7845\,\text{MeV}$

Mass difference = 0.7845 / 931 a.m.u. = 0.000842 a.m.u.

6. Calculate the Q-values of the following two reactions

(i) $^{3}_{1}H+^{2}_{1}H\rightarrow{}^{4}_{2}He+{}^{1}_{0}n$ and (ii) $^{4}_{2}He+{}^{14}_{7}N\rightarrow{}^{17}_{8}O+{}^{1}_{1}H$.

Given $M\left(^{3}_{1}H\right)=3.0169982$, $M\left(^{2}_{1}H\right)=2.0147361$, $M\left(^{4}_{2}He\right)=4.0038727$, $M\left(^{1}_{0}n\right)=1.0089832$, $M\left(^{14}_{7}N\right)=14.003074$, $M\left(^{17}_{8}O\right)=16.999133$ and $M\left(^{1}_{1}H\right)=1.007825$ all in a.m.u. Indicate also if the reaction is exoergic or endoergic.

Soln. We have $Q=\left(\text{mass of reactants}-\text{mass of products}\right)\times c^2=\Delta m.c^2$

(i) Here, $\Delta m=\left[\left\{M\left(^{3}H\right)+M\left(^{2}H\right)\right\}-\left\{M\left(^{4}He\right)+M\left(^{1}n\right)\right\}\right]$

$$= \left[\left(3.0169982 + 2.0147361\right) - \left(4.0038727 + 1.0089832\right)\right] \text{a.m.u.} = 0.0188784 \text{ a.m.u.}$$

$$\Rightarrow Q = 0.0188784 \times 931.48 \text{ MeV} = +17.57 \text{ MeV (Reaction is exoergic)}$$

(ii) Here, $\Delta m = \left[\left\{M\left({}^{4}He\right) + M\left({}^{14}N\right)\right\} - \left\{M\left({}^{17}O\right) + M\left({}^{1}H\right)\right\}\right]$

$$= \left[\left(4.003872 + 14.003074\right) - \left(16.999133 + 1.007825\right)\right] \text{a.m.u.}$$

$$= \left(18.0006946 - 18.006958\right) \text{a.m.u.} = -0.000012 \text{ a.m.u.}$$

$$= -0.000012 \times 931.48 \text{ MeV} = -0.11178 \text{ MeV}$$

So, the reaction is endoergic.

7. Assume that 4 Hydrogen nuclei are used to form a Helium nucleus in the sun to provide the total energy to it. Calculate (i) the energy released when 1 gm-atom of Hydrogen is fused to Helium, (ii) how much Hydrogen is to be converted to Helium in the Sun per second, given: mass of Hydrogen nucleus = 1.00813 a.m.u, mass of *He*-nucleus = 4.00386 a.m.u, solar constant = 1.35 kW.m^{-2}. Earth-Sun distance = 1.5×10^{8} km and there is no other loss in the energy radiated by the Sun.

Soln. (i) Mass of *H*-nucleus = 1.00813 a.m.u; Mass of *He*-nucleus = 4.00386 a.m.u.
Therefore, mass difference on fusion = (4×1.00813 – 4.00386) a.m.u. = (4.03252 – 4.00386) a.m.u. = 0.02866 a.m.u.
Therefore, energy released = 0.02866×931 MeV = 26.68 MeV. This amount is released due to fusion of 4 *H*-atom into Helium.

Therefore, energy released due to fusion of 1 gm-atom of *H*

$$= \frac{(26.68 \times 6.02 \times 10^{23})}{4.} \text{ MeV} = 40.15 \times 10^{23} \text{ MeV}$$

(ii) Energy released on fusion of 4 H-atom, $E = 0.02866 \text{ a.m.u.} \times c^2 = 0.02866 \times 1.66 \times 10^{-27} \text{ kg}$
$= 0.02866 \times 1.66 \times 10^{-27} \times (3\times10^{8})^2 \text{ J} = 4.28\times10^{-12} \text{ J}$

Therefore, energy released per *H*-atom = $(4.28\times10^{-12}/4)\text{J} = 1.07\times10^{-12}$ J

Total energy radiated per sec. from the Sun $= 1.35\times10^{3}\times4\pi\times\left(1.5\times10^{11}\right)^2 = 3.82\times10^{26}$ J

Therefore, number of *H*-atom required $= \dfrac{3.82\times10^{26}}{1.07\times10^{-12}} = 3.57\times10^{38}$

Therefore, required mass of Hydrogen $= 3.57\times10^{38}\times1.00813\times1.66\times10^{-27}$ kg $= 5.97\times10^{11}$ kg

8. Consider the nuclear reaction $N^{14}(\alpha, p)O^{17}$. Mass of neutral atoms are given to be

$${}_{1}H^{1} = 1.007825 \text{ a.m.u.}, \qquad {}_{2}He^{4} = 4.002603 \text{ a.m.u.}$$

$${}_{7}N^{14} = 14.003074 \text{ a.m.u.}, \; {}_{8}O^{17} = 16.994131 \text{ a.m.u.}$$

determine the *Q*-value of the reaction in MeV. Calculate the threshold *K.E.* of α-particle for the reaction.

Soln. Reaction ${}_{7}N^{14} + {}_{2}He^{4} \longrightarrow {}_{1}H^{1} + {}_{8}O^{17}$

$X \quad x \quad y \quad Y$

$$Q = \left(M_x + m_x - M_Y - m_y\right)c^2 = -\left(0.001279 \text{ a.m.u.}\right)c^2 = -0.001279\times931.5 \text{ MeV}$$

$$E_x^{th} = -Q\left[\frac{m_y + M_Y}{m_y + M_Y - m_x}\right] = 1.535\,494 \text{ MeV}$$

9. Complete the following Nuclear Reactions:

(i) ${}_{17}Cl^{35} + ? \rightarrow {}_{16}S^{32} + {}_2He^4$ **(Ans.** ${}_1H^1$**)** (ii) ${}_5B^{10} + ? \rightarrow {}_3Li^7 + {}_2He^4$ **(Ans.** ${}_0n^1$**)**,

(iii) ${}_3Li^6 + ? \rightarrow {}_4Be^7 + {}_0n^1$ **(Ans.** ${}_1H^2$**)** (iv) ${}_{13}Al^{27} + {}_0n^1 \rightarrow {}_{12}Mg^{27} + ?$ **(Ans.** ${}_1H^1$**)**,

(v) ${}_4Be^9 + {}_2He^4 \rightarrow ? + {}_0n^1$ **(Ans.** ${}_6C^{12}$**)**

(vi) ${}_3Li^7 +$ proton $\rightarrow \alpha$-particle $+$? **(Ans.** α-particle ${}_2He^4$**)**

(vii) Deutron + Deutron $\rightarrow$ proton + ? **(Ans.** tritium, ${}_1H^3$**)**

10. Complete the following disintegration reaction by substituting the proper nuclide or particle for the question mark in each case.

(i) ${}_{14}Si^{28}(n, p)?$ (ii) ${}_3Li^7(p, \gamma)?$ (iii) ${}_{13}Al^{27}(\alpha, p)?$ (iv) ${}_{11}Na^{23}(p, ?)\,{}_{10}Ne^{20}$

(v) ${}_{13}Al^{27}(?, \alpha)\,{}_{11}Na^{24}$ (vi) ${}_5B^{10}(?, \alpha)\,{}_3Li^7$ (vii) ${}_{11}Na^{23}(d, ?)\,{}_{13}Mg^{24}$

Soln. (i) ${}_{13}Al^{28}$ (ii) ${}_4Be^8$ (iii) ${}_{14}Si^{30}$ (iv) ${}_2He^4$ (v) ${}_0n^1$ (vi) ${}_0n^1$ (vii) ${}_0n^1$

11. Complete the following Reactions

(i) $H^1(n, \gamma)$? (ii) $Li^7(p, ?)\,Be^7$ (iii) $N^{14}(?, p)\,O^{17}$ (iv) $?(n, p)\,Na^{24}$

Soln. (i) ${}_1H^2$, (ii) ${}_0n^1$, (iii) ${}_2He^4$ and (iv) ${}_{12}Mg^{24}$

12. An α-particle with *K.E.* $T_\alpha = 7.0$ MeV is scattered elastically by an initially stationary Li^6 nucleus. Find the *K.E.* of the recoil nuclers if the angle of divergence of the two particles is $\theta = 60°$.

Soln. Initial momentum of α-particle is $\sqrt{2mT_\alpha}\,\hat{i}$

Final momenta are respectively $\vec{p}_\alpha$ and $\vec{p}_{Li}$.

Conservation of momentum reads

$$\vec{p}_\alpha + \vec{p}_{Li} = \sqrt{2mT_\alpha}\,\hat{i} \quad \Rightarrow \quad p_\alpha^2 + p_{Li}^2 + 2p_\alpha p_{Li}\cos\theta = 2mT_\alpha \qquad ...(1)$$

Where θ is the angle between $\vec{p}_\alpha$ and $\vec{p}_{Li}$

Energy conservation: $\frac{p_\alpha^2}{2m} + \frac{p_{Li}^2}{2M} = T_\alpha \quad \Rightarrow \; p_\alpha^2 + \frac{m}{M}p_{Li}^2 = 2mT_\alpha \qquad ...(2)$

Where, m = mass of α, M = mass of Li^6

Subtract equation (2) from equation (1) we get. $p_{Li}\left[\left(1 - \frac{m}{M}\right)p_{Li} + 2p_\alpha\cos\theta\right] = 0, \qquad p_{Li} \neq 0$

$$\Rightarrow \; p_\alpha = -\frac{1}{2}\left(1 - \frac{m}{M}\right)p_{Li}\sec\theta$$

Now p_α, p_{Li} are both positive number (bring magnitudes of vectors) we must have $-1 \le \cos\theta < 0$ if $m < M$

Thus, we write, $\frac{p_{Li}^2}{2M}\left[1+\frac{M}{4m}\left(1-\frac{m}{M}\right)^2 \sec^2\theta\right] = T_\alpha$

Hence, recoil energy of Li nucleus is $\frac{p_{Li}^2}{2M} = \frac{T_\alpha}{1+\frac{(M-m)^2}{4mM}\sec^2\theta}$

Putting, $\theta = 120°$, we get recoil energy of Li = 6 MeV.

13. For each of the following decays state a conservation law that forbids it :

$n \to p + e^-$

$n \to \pi^+ + e^-$

$n \to p + \pi^-$

$n \to p + \gamma$

Soln. $n \to p + e^-$: conservation of angular momentum and conservation of lepton number are both violated.

$n \to \pi^+ + e^-$: conservation of baryon number and conservation lepton number are both violated.

$n \to p + \pi^-$: conservation of energy is violated (as mass of $\pi^- \approx 140$ MeV/c^2).

$n \to p + \gamma$: conservation of electric charge is violated.

14. What amount of heat is liberated during the formation of one gram of He^4 from Deuterium H^2? What mass of the coal with calorific value of 30 kJ/g is thermally equivalent to the magnitude obtained?

Soln. Reaction is $H^2 + H^2 \to He^4 + Q$

$Q = 2\Delta_{H^2} - \Delta_{He^4} = (0.02820 - 0.00260)c^2 = (0.02560 \text{ a.m.u.})c^2 = 23.8$ MeV

Hence, the energy released in the 1g of He^4 is $\frac{6.023\times10^{23}}{4} \times 23.8 \times 16.02 \times 10^{-13}$ Joule $= 5.75 \times 10^8$ kJ

This energy can be derived from $\frac{5.75\times10^8}{30000}$ kg $= 1.9 \times 10^4$ kg of coal

15. Making use of the tables of atomic masses, determine the energies of the following reaction.

(a) $Li^7(p, n)\, Be^7$ (b) $Be^9\ (n, r)\ Be^{10}$ (c) $Li^7\ (\alpha, n) B^{10}$ (d) $O^{16}(d, \alpha) N^{14}$

Soln. **(a)** $Li^7\ (p, n)\ Be^7$:

$Q = \left(M_{Li^7} + M_{Be^7}\right)c^2 + \left(M_p - M_n\right)c^2 = \left(\Delta_{Li^7} - \Delta_{Be^7}\right)c^2 + \Delta_p - \Delta_n$

$= [0.01601 + 0.00783 - 0.01693 - 0.00867]$ a.m.u. $\times c^2 = -1.64$ MeV.

(b) $Be^9\ (n, \gamma)\, Be^{10}$: Mass of γ is taken as zero.

$Q = \left(M_{Be^9} + M_n - M_{Be^{10}}\right)c^2 = \left(\Delta_{Be^9} + \Delta_n - \Delta_{Be^{10}}\right)c^2 = (0.01219 + 0.00867 - 0.01354)$ a.m.u. $\times c^2$

$= 6.81$ MeV.

(c) $Li^7(\alpha, n)B^{10}$:

$Q = \left(\Delta_{Li^7} + \Delta_\alpha - \Delta_n - \Delta_{B^{10}}\right)$ = (0.01601 + 0.00260 – 0.00867 – 0.01294) a.m.u. × c^2 = – 2.79 MeV

(d) $O^{16}(d, \alpha)N^{14}$: $Q = \left(\Delta_{O^{16}} + \Delta_d - \Delta_\alpha - \Delta_{N^{14}}\right)c^2$

= (–0.00509 + 0.01410 – 0.00260 – 0.00307) a.m.u. × c^2 = 3.11 MeV

16. Find the velocity with which the products of the reaction $B^{10}(n, \alpha)Li^7$ come apart; the reaction proceeds via interaction of very slow neutrons with stationary Boron nuclei.

Soln. We have : $B^{10}(n, \alpha)Li^7$, $Q = \left(\Delta_{B^{10}} + \Delta_n - \Delta_\alpha - \Delta_{Li^7}\right)c^2$

= (0.01294 + 0.00867 – 0.00260 – 6.01601) a.m.u. × c^2= 2.79 MeV.

Since the incident neutron is very slow and B^{10} is stationary, the final total momentum must also be Zero. So the reaction products must emerge in opposite direction. If their speeds are respectively, V_α and V_{Li} then $4V_\alpha = 7V_{Li}$ and $\frac{1}{2}\left(4V_\alpha^2 + 7V_{Li}^2\right)\times 1.672\times 10^{-24} = 2.79\times 1.602\times 10^{-6}$

So, $\frac{1}{2}\times 4V_\alpha^2\left(1+\frac{4}{7}\right) = 2.70\times 10^{18}\,\text{cm}^2/\text{s}^2 \Rightarrow V_\alpha = 9.27\times 10^6\,\text{m/sec}$

$\Rightarrow$ V_{Li} = 5.3 × 10^6 m/sec.

17. Protons striking a stationary Lithium target activate a reaction $Li^7(p, n)Be^7$. At what value of the proton *K.E.* can the resulting neutron be stationary?

Soln. Q = –1.64 MeV.

Conservation of momentum: $p_p = p_{Be}$ (since initial *Li* and final neutron both are at rest)

$\Rightarrow$ $\frac{p_p^2}{2m_p} = \frac{p_{Be}^2}{2m_{Li}} + 1.64$ $\Rightarrow$ $\frac{p_p^2}{2m_p}\left(1 - \frac{m_p}{m_{Be}}\right) = 1.64$

Hence, $T_p = \frac{p_p^2}{2m_p} = \frac{7}{6}\times 1.64\ MeV = 1.91\ MeV$

CAREER ENDEAVOUR

18. A beam of proton (of a certain energy) equivalent to a current of 1.6 mA is incident uniformly on a ^{56}F target containing 10^{22} atom per m². So, that the following reaction take places, $p + {}^{56}Fe \longrightarrow n + {}^{53}Co$. If cross-section for the reaction is 1 barn, calculate the number of neutrons produced per sec.

Soln. $^{56}Fe + p \longrightarrow {}^{53}Co + n$ Given : i =1.6 mA =1.6×10^{-3} A

$\Rightarrow$ $q = it$ $\Rightarrow \frac{q}{t} = i$ Since, $q = ne$ $\Rightarrow$ $\frac{ne}{t} = i$

Number of incident particle per unit time is $N_0 = \frac{n}{t} = \frac{i}{e} = \frac{1.6\times 10^{-3}}{1.6\times 10^{-19}} = 10^{16}/\text{sec}$

No. of particle which interact with target is $N_0 - N = N_0 - N_0 e^{-\sigma nx} = N_0\left\{1 - e^{-\sigma nx}\right\}$

where, $nx = 10^{22}$ atom per m².

$\Rightarrow N_0 - N = 10^{16}\left\{1 - e^{-\left\{10^{-6}\right\}}\right\} = 10^{10}$ = Number of neutrons produced per second.

19. A 16 μA beam of a particle having cross-sectional area of 10^{-4} m^2 is incident on a *Rh* target of thickness 1 μm. This produces neutrons through the reaction $\alpha + {}^{100}Rh \longrightarrow Pd + 3n$

(1) Number of α–particle hitting the target per second is:

(a) $0.5\times10^{+14}$ (b) 1×10^{14} (c) 2×10^{20} (d) 4×10^{20}

Soln. $\frac{n}{t} = \frac{i}{2e} = \frac{16\times10^{-6}}{2\times1.6\times10^{-19}} = 0.5\times10^{14}$

20. Consider the decay process $\tau^- \to \pi^- + \nu_\tau$ in the rest frame of the τ^-. The masses of τ^-, π^- and ν_τ are μ_τ, μ_π and zero respectively. Find the energy and velocity of π^-.

Soln. Kinetic energy = $-Q\left[\frac{\text{Sum of masses of all particles involved in the reaction}}{2\times\text{mass of target}}\right]$

Where $Q = [m_i - m_f]c^2 = [m_\tau - m_\pi - 0]c^2$

Kinetic energy = $-(m_\tau - m_\pi)c^2\left[\frac{m_\tau + m_\pi}{2m_\tau}\right] = \left[\frac{-m_\tau^2 - m_\tau m_\pi + m_\pi m_\tau + m_\pi^2}{2m_\tau}\right]c^2 = \frac{(-m_\tau^2 + m_\pi^2)}{2m_\tau}c^2$

The energy of π^- is

E_{th} = Kinetic energy + rest mass energy

$= \frac{[-m_\tau^2 + m_\pi^2]c^2}{2m_\tau} + m_\tau c^2 = \frac{(m_\tau^2 + m_\pi^2)c^2}{2m_\tau}$

21. At a centre-of-mass energy of 5 MeV, the phase describing the elastic scattering of a neutron by a certain nucleus has the following values, $\delta_0 = 30°$, $\delta_1 = 10^0$. Assuming all other phase shifts to be negligible, plot $d\sigma/d\Omega$ as a function of scattering angle. Explicitly calculate $d\sigma/d\Omega$ at 30°, 45° and 90°. What is the total cross section σ?

Soln. The differential cross section is given $\frac{d\sigma}{d\Omega} = \frac{1}{k^2}\left|\sum_{\ell=0}^{\infty}(2\ell+1)e^{i\delta_\ell}\sin\delta_\ell P_\ell(\cos\theta)\right|^2$

Supposing only the first and second terms are important, we have

$$\frac{d\sigma}{d\Omega} \approx \frac{1}{k^2}\left|e^{i\delta_0}\sin\delta_0 + 3e^{i\delta_1}\sin\delta_1\cos\theta\right|^2$$

$$= \frac{1}{k^2}\left[\left(\cos\delta_0\sin\delta_0 + 3\cos\delta_1\sin\delta_1\cos\theta\right) + i\left(\sin^2\delta_0 + 3\sin^2\delta_1\cos\delta\right)\right]^2$$

$$= \frac{1}{k^2}\left[\sin^2\delta_0 + 9\sin^2\delta_1\cos^2\theta + 6\sin\delta_0\sin\delta_1\cos\left(\delta_1 - \delta_0\right)\cos\theta\right]^2$$

$$= \frac{1}{k^2}\left[0.25 + 0.27\cos^2\theta + 0.49\cos\theta\right]$$

where k is the wave number of the incident neutron in the centre of mass frame. Assume that the mass of the nucleus is far larger than that of the neutron m_n. Then

$$k^2 \approx \frac{2m_n E}{\hbar^2} = \frac{2m_n c^2 E}{(hc)^2} = \frac{2\times 938\times 5}{197^2\times 10^{-30}} = 2.4\times 10^{29}\,\text{m}^{-2} = 2.4\times 10^{25}\,\text{m}^{-2}$$

0	0^0	30^0	45^0	90^0	180^0
$k^2\frac{d\sigma}{d\Omega}$	1	0.88	0.73	0.25	0
$\frac{d\sigma}{d\Omega}\times 10^{26}\,(\text{cm}^2)$	4.2	3.7	3.0	1.0	0

The total cross section is:

$$\sigma = \int \frac{d\sigma}{d\Omega} = \frac{2\pi}{k^2}\int_0^{\pi}\left(0.25 + 0.49\cos\theta + 0.27\cos^2\theta\right)\sin\theta\, d\theta$$

$$= \frac{4\pi}{k^2}\left(0.25 + \frac{1}{3}\times 0.27\right) = 1.78\times 10^{-25}\,\text{cm}^2 \approx 0.18\ \text{barn}$$

22. Neutrons of 1000 eV kinetic energy are incident on a target composed of carbon. If the inelastic cross section is 400×10^{-24} cm^2, what upper and lower limits can you place on the elastic scattering cross section?

Soln. At 1 keV kinetic energy, only s-wave scattering is involved. The phase shift δ must have a positive imaginary part for inelastic process to take place. The elastic and inelastic cross sections are respectively given by

$$\sigma_e = \pi\lambda^2\left|e^{2i\delta} - 1\right|^2,\ \sigma_{in} = \pi\lambda^2\left(1 - \left|e^{2i\delta}\right|^2\right)$$

The reduced mass of the system is $\mu = \frac{m_n m_e}{m_e + m_n} \approx \frac{12}{13}m_n$

For E = 1000 eV, $\lambda\!\!\!^{-} = \frac{\hbar}{\sqrt{2\mu E}} = \frac{\hbar c}{\sqrt{2\mu c^2 E}} = \frac{197}{\sqrt{2\times\frac{12}{13}\times 940\times 10^{-3}}}\,\text{m} = 150\ \text{fm}$

$$\pi\lambda\!\!\!^{-2} = 707\times 10^{-24}\ \text{cm}^2$$

As, $1 - \left|e^{2i\delta}\right|^2 = \frac{\sigma_{in}}{\pi\lambda\!\!\!^{-2}} = \frac{400\times 10^{-24}}{707\times 10^{-24}} = 0.566$

We have, $\left|e^{2i\delta}\right| = \sqrt{1 - 0.566} = 0.659 \Rightarrow e^{2i\delta} = \pm 0.659$

Hence, the elastic cross section $\sigma_e = \pi\lambda\!\!\!^{-2}\left|e^{2i\delta} - 1\right|^2$

has maximum and minimum values $(\sigma_e)_{\max} = 707\times 10^{-24}(-0.659 - 1)^2 = 1946\times 10^{-24}\ \text{cm}^2$

$$(\sigma_e)_{\min} = 707\times 10^{-24}(-0.659 - 1)^2 = 82\times 10^{-24}\ \text{cm}^2$$

23. Disregarding nucleon spin, set a lower bound on the elastic center of mass proton-neutron forward differential cross-section.

Soln. The forward p–n differential cross section is given by

$$\left.\frac{d\sigma}{d\Omega}\right|_{0^\circ} = |f(0)|^2 \geq |\text{Im}\, f(0)|^2 = \left(\frac{k}{4\pi}\sigma_t\right)^2$$

where the relation between $\text{Im}\,f(0)$ and σ_t is given by the optical theorem. As $k = p/h$ we have

$$\left.\frac{d\sigma}{d\Omega}\right|_{0^\circ} \geq \left(\frac{pc}{4\pi hc}\sigma_t\right)^2 = \left(\frac{10^4 \times 40 \times 10^{-27}}{4\pi \times 1.97 \times 10^{-11}}\right)^2 \text{m}^2 = 2.6 \times 10^{-24}\ \text{cm}^2 = 2.6\ \text{barn}$$

24. If hydrogen atom is bombarded by energetic electrons, it will emit **[JEST 2014]**

(a) K_α X-rays (b) β-rays (c) Neutrons (d) None of the above

Soln. In given option (a), X-rays are produced by bombarding electrons on high atomic, high melting point materials like *Mo*, *Cu* etc.

In option (b), β-rays are used in radioactive decay.

In option (c), Neutrons have larger energy than electrons, so electron can not emit them.

While according to question, if hydrogen atom is bombarded by energy electrons, it will emit neutrinos.

Correct option is (d)

25. The binding energy per uncleon of ^{235}U is 7.6 MeV. The ^{235}U nucleus undergoes fission to produce two fragments, both having binding energy per nucleon 8.5 MeV. The energy released, in Joules, from the complete fission of 1 kg of ^{235}U is therefore, **[TIFR 2010]**

(a) 8000 (b) 10^{35} (c) 450 (d) 20000

(e) 8.7×10^{13} (f) 5.0×10^{8}

Soln. $^{235}\text{U} \longrightarrow \text{X} + \text{Y}$

Here no β- decay is taking place so number of proton and neutron before decay is equal to number of proton and neutron after decay.

Therefore, $\text{M}\left({}^{\text{A}}_{\text{Z}}\text{X}\right)c^2 = 2m_p c^2 + (\text{A} - \text{Z})m_n c^2 - \text{B.E.}$

So, using law of conservation of energy, $-7.6 \times 235 = -8.5 \times 235 + x$

$\Rightarrow\ x = (0.9 \times 235)\ \text{MeV} \ \Rightarrow\ x = 211.5\ \text{MeV}$

Therefore, 1 atom of ^{235}U will release 211.5 MeV energy.

235g will contains 6.022×10^{23} atoms

So, 1 g will contains $\dfrac{6.022 \times 10^{23}}{235}$ atoms

Therefore, 1 kg will contains $\dfrac{6.022 \times 10^{23}}{235} \times 1000$ atoms

So, total energy will release $= \dfrac{6.022 \times 10^{26}}{235} \times 211.5\ \text{MeV} = 5.42 \times 10^{26}\ \text{MeV}.$

So, energy release in joule $= 5.42 \times 10^{26} \times 1.6 \times 10^{-19} \times 10^{6} = 8.67 \times 10^{13}\ \text{J}.$

Correct option is (e)

26. A fast-moving ^{14}N nucleus collides with an α particle at rest in the laboratory frame, giving rise to the reaction

$$^{14}N + \alpha \rightarrow {}^{17}O + p \qquad \textbf{[TIFR 2011]}$$

Given the masses 14.00307 a.m.u. and 16.99913 a.m.u. for ^{14}N and ^{17}O nuclei respectively, and 4.00260 a.m.u. and 1.00783 a.m.u. for α and p respectively, the minimum kinetic energy in the laboratory frame of the ^{14}N nucleus must be

(a) 4.20 MeV (b) 1.20 MeV (c) 5.41 MeV (d) 1.55 MeV

Soln. Given reaction, $(X)\ (x)\quad (Y)\quad (y)$

$$^{14}N + \alpha \longrightarrow {}^{17}O + p$$

Now, formula for Q-value

$$Q = \{(\text{mass of reactants}) - (\text{mass of products})\} \times 931.48 \text{ MeV}$$

$$Q = \{(m(^{14}N) + m(\alpha)) - (m(^{17}O) + m(p))\} \times 931.48 \text{ MeV}$$

$$= \{(14.00307 + 4.00260) - (16.99913 + 1.00783)\} \times 931.48 \text{ MeV}$$

$$= -0.00129 \times 931.48 = -1.2016092 \text{ MeV}$$

Therefore, minimum energy,

$$E_{th}^{X} = \frac{-Q(m_y + M_Y)}{(m_y + M_Y - m_x)} = -Q\left(\frac{1.00783 + 16.99913}{1.00783 + 16.99913 - 14.00307}\right) \text{MeV}$$

$$= 1.2016 \times \frac{18.00693}{4.00389} = 5.41 \text{ MeV}$$

Correct option is (c)

27. In a nuclear reactor, Plutonium ($^{239}_{94}Pu$) is used as fuel, releasing energy by its fission into isotopes of Barium ($^{146}_{54}Ba$) and Strontium ($^{91}_{38}Sr$) through the reaction **[TIFR 2012]**

$$^{239}_{94}Pu + {}^{1}_{0}n \longrightarrow {}^{146}_{56}Ba + {}^{91}_{38}Sr + 3 \times {}^{1}_{0}n$$

The binding energy ($B.E.$) per nucleon of each of these nuclides is given in the table below:

Nuclide	$^{239}_{94}Pu$	$^{146}_{54}Ba$	$^{91}_{38}Sr$
B.E. per nucleon (MeV)	7.6	8.2	8.6

Using this information, one can estimate the number of such fission reactions per second in a 100 MW reactor as

(a) 3.9×10^{18} (b) 7.8×10^{18} (c) 5.2×10^{19} (d) 5.2×10^{18}

(e) 8.9×10^{17}

Soln. $^{239}_{94}\text{Pu} + {}^{1}_{0}\text{n} \longrightarrow {}^{146}_{56}\text{Ba} + {}^{91}_{38}\text{Sr} + 3 \times {}^{1}_{0}\text{n}$

$$M\left({}^{A}_{Z}X\right)c^2 = Z m_p c^2 + (A - Z) m_n c^2 - \text{B.E.}$$

So, according to law of conservation of energy

$$M\left({}^{239}_{94}\text{Pu}\right)c^2 + m\left({}^{1}_{0}n\right)c^2 = M\left({}^{146}_{56}\text{Ba}\right)c^2 + M\left({}^{91}_{38}\text{Sr}\right)c^2 + 3m\left({}^{1}_{0}n\right)c^2$$

$$94 m_p c^2 + 145 m_n c^2 - 7.6 \times 239 + m_n c^2$$

$$= 56 m_p c^2 + 90 m_n c^2 - 8.2 \times 146 + 38 m_p c^2 + 53 m_n c^2 - 91 \times 8.6 + 3 m_n c^2$$

$$\Rightarrow 94 m_n c^2 + 146 m_n c^2 - 1816.4 = 94 m_n c^2 + 146 m_n c^2 - 1197.2 - 782.6 + x$$

$$x = (1197.2 + 782.6 - 1816.4) \text{ MeV}$$

$$x = 163.4 \text{ MeV}$$

Therefore, 1 reaction 163.41 MeV energy will release

To find number of reaction per second to produce 100 MW energy,

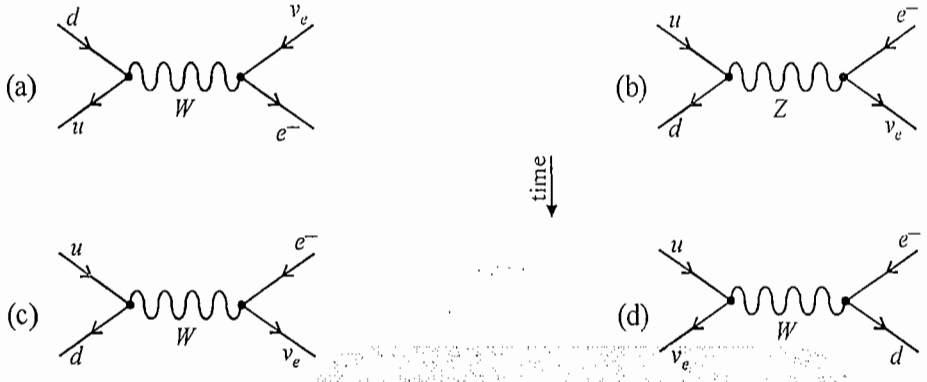
$$n=\frac{100\times10^{6}}{163.41\times10^{6}\times1.6\times10^{-19}}\Rightarrow n=\frac{100\times10^{19}}{1.6\times163.41}\Rightarrow n=3.83\times10^{18}.$$

Correct option is (a)

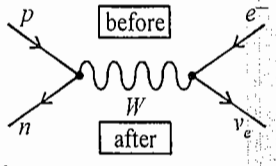
28. The process of electron capture **[TIFR 2013]**

$$p+e^{-}\rightarrow n+\nu_e$$

takes place at the quark level through the Feynman diagram

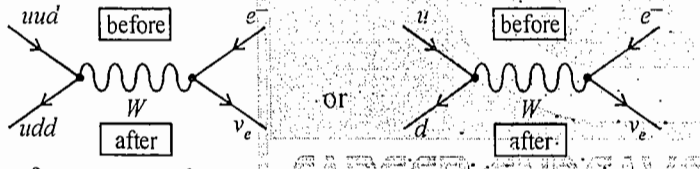


Soln. The process of electron capture, $p+e^{-}\longrightarrow n+\nu_e$ can be represented in Feynman's diagram as



In this process proton turns into a neutron by trapping an electron. The exchange particle W leaves a neutrino after the reaction.

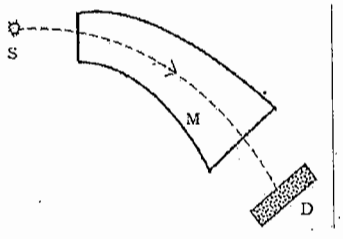
In the quark level, the Feynman diagram can be represented as



Since for proton and neutron, one u and one d are common.

Correct option is (c)

29. In a beta decay experiment, an electromagnetic M and a detector D are used to measure the energy of electrons (β^{-}), as shown in the figure. **[TIFR 2014]**



The detector D is capable of detecting either electons (β^{-}) or positrons (β^{+}). Now the β^{-} source is replaced with a β^{+} source, and we would like to measure the energy of the positrons (β^{+}) using the same setup. Which of the following is correct?

(a) This can be done quite easily, if the polarity of current in the coils of the electromagnet is reversed.

(b) This is can done trivially, without changing anything, since the detector D can detect either β^{-} or β^{+}.

(c) There is no way to do this with the given set up, since β^+ will have to be converted into β^-, which is obviously not possible.

(d) This cannot be done since the magnet does not have a symmetric shape.

Soln. If the β^- source is replaced with a β^+ source. Then as the charge on the source is reversing, we need to reverse the polarity of the current in the coils of the electromagnet. So, that we can detect them by using same detector D.

Correct option is (a)

30. Consider a process in which atoms of Actinium-226 $\left({}^{226}_{89}Ac\right)$ get converted to atoms of Radium-226 $\left({}^{226}_{88}Ra\right)$ and the yield of energy is 0.64 MeV per atom. This occurs through **[TIFR 2016]**

(a) Both $p \to n + e^+ + \nu_e$ and $p + e^- \to n + \nu_e$

(b) Both $p \to n + e^+ + \nu_e$ and $n \to p + e^- + \overline{\nu}_e$

(c) Only $p \to n + e^+ + \nu_e$

(d) Only $p + e^- \to n + \nu_e$

Soln. Therefore, ${}^{226}_{89}\text{Ac} \longrightarrow {}^{226}_{88}\text{Ra}$, so here number of proton is decreasing by 1.

So, $n \longrightarrow P + e^- + \overline{\nu}_e$ can't possible because

$$Q_{\text{electron capture}} = \left[m\left({}^{226}_{89}\text{Ac}\right) - m\left({}^{226}_{88}\text{Ra}\right)\right]c^2$$

$$Q_{\beta^+}\text{decay} = \left[m\left({}^{226}_{89}\text{Ac}\right) - m\left({}^{226}_{88}\text{Ra}\right) - 2m_e\right]c^2$$

$$Q_{\text{electron capture}} = Q_{\beta^+}\text{decay} + 2m_e c^2$$

$$Q_{\text{electron capture}} = Q_{\beta^+}\text{decay} + 1.02\ \text{MeV}$$

Here if $Q_{\beta^+}\text{decay} > 0$, then $Q_{\text{electron capture}} > 0$; Here only energy yield = 0.64 MeV.

So it is possible when $Q_{\beta^+}\text{decay}$ is negative means Q_{β^+} cannot be possible so only electron capture can be possible.

Correct option is (d)

31. A deuteron of mass M and binding energy B is struck by a gamma ray photon of energy E_γ, and is observed to disintegrate into a neutron and a proton. If $B \ll Mc^2$, the minimum value of E_γ must be

(a) $2B + \dfrac{B^2}{2Mc^2}$ (b) $B + \dfrac{B^2}{Mc^2}$ (c) $\dfrac{1}{2}\left(3B + \dfrac{B^2}{Mc^2}\right)$ (d) $\dfrac{1}{2}\left(2B + \dfrac{B^2}{Mc^2}\right)$ **[TIFR 2017]**

Soln. ${}^2_1H + \gamma \to n + p$; so minimum threshold energy in C.M. frame products have K.E. = 0.

So, in C.M. frame, momentum of $\gamma = \frac{E}{c}$, so momentum of ${}^2_1H = -\frac{E}{c}$.

Therefore, energy ${}^2_1H = \sqrt{M^2c^4 + p^2c^2} = \sqrt{M^2c^4 + E^2}$

So according to law of conservation of energy

$$\sqrt{M^2c^4 + E^2} + E = m_n c^2 + m_p c^2$$

$$\sqrt{M^2c^4+E^2}=\left(m_n+m_p\right)c^2-E$$

So, $M^2c^4+E^2=\left(m_n+m_p\right)^2c^4+E^2-2E\left(m_n+m_p\right)c^2$

$$E=\frac{\left(m_n+m_p\right)^2c^4-M^2c^4}{2\left(m_n+m_p\right)c^2} \qquad \ldots(A)$$

$$Mc^2=m_pc^2+m_nc^2-B \Rightarrow \frac{Mc^2+B}{c^2}=m_p+m_n$$

Put in equation (A), so

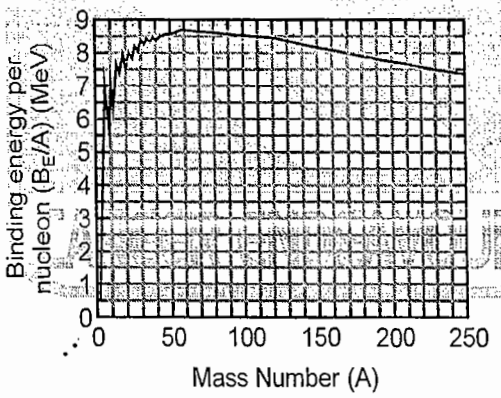
$$E=\frac{\left(\frac{Mc^2+B}{c^2}\right)c^4-M^2c^4}{2\left(\frac{Mc^2+B}{c^2}\right)c^2}=\frac{\left(Mc^2+B\right)^2-M^2c^4}{2Mc^2\left(1+\frac{B}{Mc^2}\right)}$$

Therefore, $B \ll Mc^2$ so $\frac{B}{Mc^2}\to 0$; $E=\frac{\left(Mc^2+B\right)^2-M^2c^4}{2Mc^2}$

$$E=\frac{M^2c^4+B^2+2Mc^2B-M^2c^4}{2Mc^2} \Rightarrow E=\frac{1}{2}\left(2B+\frac{B^2}{Mc^2}\right).$$

Correct option is (d)

32. In a theoretical model of the nucleus, the binding energy per nucleon was predicted as shown in the figure below. **[TIFR 2017]**



If a nucleus of mass number A = 240 undergoes a symmetric fission to two daughter nuclei each of mass number A = 120, write down the amount of energy released in this process, in units of MeV, using this theoretical model.

Soln. $^{240}X \longrightarrow {}^{120}Y + {}^{120}Z$

Therefore, number β- decay is taking place, so number of protons no initial will be also equal to number of protons final.

Therefore, also $M\left({}^{A}_{Z}X\right)c^2 = Zm_pc^2 + (A-Z)\,m_nc^2 - \text{B.E.}$

So, according to law of conservation of energy $M\left(^{240}X\right)c^2 = M\left(^{120}Y\right)c^2 + M\left(^{120}Z\right)c^2 + x$

Due to equal number of proton and neutron in reactants and products their corresponding energy will be cancel out so, $-7.6\times240 = -8.5\times120 - 8.5\times120 + x$.

$\Rightarrow \quad x = (8.5\times240 - 7.5\times240)\ \text{MeV}$

$x = 240$ MeV

So, energy release will be 240 MeV.

Correct answer is (240 MeV)

33. Cosmic ray muons generated at the top of the Earth's atmosphere decay according to the radioactive decay law **[TIFR 2014]**

$$N(t) = N(0)\exp\left(-\frac{0.693t}{T_{1/2}}\right)$$

where N(t) is the number of muons at time t, and $T_{1/2}$ = 1.52μs is the proper half-life of the muon. Immediately after generation, most of these muons shoot down towards the Eearth's surface. Some of these muons decay on the way, but their interaction with the atmosphere is negligible.

An observer on the top of a mountain of height 2.0 km above mean sea level detects muons with the speed 0.98c over a period of time and counts 1000 muons. The number of muons of the same speed detected by an observer at mean sea level in the same period of time would be

(a) 232 (b) 539 (c) 839 (d) 983

Soln. $N = N_0 e^{-\left(-\frac{0.693}{T_{1/2}}t\right)}$

Here, $T_{1/2}$ for observer is

$$\frac{T_{1/2}}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1.52\times10^{-6}}{\sqrt{1-(0.98)^2}} = 7.638\ \mu\text{sec}$$

then $N_0 = 1000$.

$T = \frac{D}{V} \Rightarrow T = \frac{2km}{0.98\times3\times10^8}$; $T = 6.8\times10^{-6}$ sec it will take this amount of time, then

$$N = 1000\exp\left[-\frac{0.693\times6.8\times10^{-6}}{7.638\times10^{-6}}\right]$$

$N = 1000\times0.53944$

So, $N = 539.44$

Correct option is (b)

34. A lead container contains 1 gm of a $^{60}_{27}Co$ radioactive source. It is known that a $^{60}_{27}Co$ nucleus emits a β particle of energy 316 KeV followed by two γ emissions 1173 and 1333 KeV respectively. Which of the following experimental methods would be the best way to determine the lifetime of the $^{60}_{27}Co$ source? **[TIFR 2010]**

(a) Measure the change in temperature of the source

(b) Measure the weight of the source now and again after one year

(c) Measure the recoil momentum of the nucleus during β emission

(d) Measure the number of γ photons emitted by this source

Soln. $N = N_0 e^{-\lambda t}$ ... (1)

we can find number of particle at $t = 0$, given initially mass $= 1\ g$ and also find how many particles has decayed by weighing after one year and by this information can also find remaining undecayed particle.

By equation (1), we can find λ.

So, life time $= \frac{1}{\lambda}$ can be determined.

Correct option is (b)

35. The Weizäcker semi-empirical mass formula for an odd nucleus with z protons and A nucleons may be written as $M(Z, A) = \alpha_1 A + \alpha_2 A^{2/3} / \alpha_3 A + \alpha_4 Z^2$ where the α_i are constant independent of Z, A. For a given A, if Z_A is the number of photons of the most stable isobar, the total energy released when an unstable nuclide undergoes a single β^- decay to (Z_A, A) is **[TIFR 2016]**

(a) α_3 (b) α_4 (c) $\alpha_4 - \alpha_3$ (d) $\alpha_1 + \alpha_2$

Soln. To find most stable nuclei of isobar $\frac{\partial B.E.}{\partial Z} = 0$, So, $B.E. = Z m_p c^2 + (A - Z) m_n c^2 - M(Z, A)$.

$$B.E. = Z m_p c^2 + (A-Z) m_n c^2 - \alpha_1 A c^2 - \alpha_2 A^{2/3} c^2 - \alpha_3 Z c^2 - \alpha_4 Z^2 c^2$$

$$\frac{\partial B.E.}{\partial Z} = m_p c^2 - m_n c^2 - \alpha_3 c^2 - 2\alpha_4 Z c^2 = 0$$

$$\Rightarrow \quad 2\alpha_4 Z c^2 = m_p c^2 - m_n c^2 - \alpha_3 c^2$$

$$\therefore \quad Z = \frac{m_p - m_n - \alpha_3}{2\alpha_4}$$

$$\therefore \quad m_p \approx m_n$$

So, $Z = -\frac{\alpha_3}{2\alpha_4}$; therefore, $Z_A = -\frac{\alpha_3}{2\alpha_4}$

$$\therefore \quad M(Z_A - 1, A) \rightarrow M(Z_A, A) + e^-$$

Energy release will be $E = M(Z_A - 1, A)c^2 - M(Z_A, A)c^2$

$$E = \left\{\alpha_1 A + \alpha_2 A^{2/3} + \alpha_3 (Z_A - 1) + \alpha_4 (Z_A - 1)^2 - \alpha_1 A - \alpha_2 A^{2/3} - \alpha_3 Z_A - \alpha_4 Z_A^2\right\} c^2$$

$$E = \left\{\alpha_3 Z_A - \alpha_3 + \alpha_4 Z_A^2 + \alpha_4 - 2\alpha_4 Z_A - \alpha_3 Z_A - \alpha_4 Z_A^2\right\} c^2$$

$$E = \left\{-\alpha_3 + \alpha_4 - 2\alpha_4 \times \left(-\frac{\alpha_3}{2\alpha_4}\right)\right\} c^2 ; \; E = \{-\alpha_3 + \alpha_4 + \alpha_3\} c^2 ; \; E = +\alpha_4 c^2$$

So most nearest answer will be option (d)

Correct option is (d)

36. A standard radioactive source is known to decay by emission of γ rays. The source is provided to a student in a thick sealed capsule of unbreakable plastic and she is asked to find out the half-life. Which of the following would be the most useful advice to the student? **[TIFR 2014]**

(a) The half-life cannot be measured because the initial concentration of the source is not given.

(b) Mount the source in front of a gamma ray detector and count the number of photons detected in one hour.

(c) Measure the mass of the source at different times with an accurate balance having a least count of 1 mg. Plot these values on a curve and fit it with an exponential decay law.

(d) Mount the source in front of a gamma ray detector and count the number of photons detected in a specific time interval. Repeat this experiment at different times and not how the count changes.

Soln. If we will count at different time then we will come to know about activity at different time and from the information can find λ and from λ can find $t_{1/2}$.

$A = A_0 e^{-\lambda t}$ and $t_{1/2} = \dfrac{\ln 2}{\lambda}$.

Correct answer is (d)

37. Number of fission per second in a 100 MW uranium reactor is

(a) 10^6 (b) 10^{12} (c) 10^{18} (d) 10^{24}

Soln. Each fission of uranium nucleus releases about 200 MeV = 320×10^{-13} J. So the number of fissions per second in a 100 MW reactor is

$$N = \frac{100 \times 10^6}{320 \times 10^{-13}} = 3 \times 10^{18}$$

Correct option is (c)

38. A deuteron of mass M and binding energy $B(B \ll Mc^2)$ is disintegrated into a neutron and a proton by a gamma ray of energy E_γ. Find to lowest order in B/Mc^2, the minimum value of $(E_\gamma - B)$ for which the reaction can occur.

(a) $\dfrac{B^2}{Mc^2}$ (b) $\dfrac{B^2}{\frac{5}{2}Mc^2}$ (c) $\dfrac{B^4}{M^2c^4}$ (d) $\dfrac{B^2}{2Mc^2}$

Soln. In the disintegration of the deuteron, $E_\gamma - B$ is smallest when E_γ is at threshold, at which the final particles are stationary in the center of mass system. In this case the energy of the incident photon in the center of mass system $= E_\gamma$.

In centre of mass frame, net momentum is zero.

So, $|p_\gamma| = |p_M|$ and also $E_\gamma = p_\gamma c$

So, initial energy of the system is,

$$E_i = E_\gamma + \sqrt{M^2c^4 + p_\gamma^2 c^2} = E_\gamma + \sqrt{M^2c^4 + E_\gamma^2} \quad \ldots(1)$$

and final energy of the system is

$$E_f = m_n c^2 + m_p c^2 \quad \ldots(2)$$

So, from law of conservation of energy

$$E_\gamma + \sqrt{M^2c^4 + p_\gamma^2 c^2} = m_n c^2 + m_p c^2$$

$$\Rightarrow \left(m_n c^2 + m_p c^2\right) - E_\gamma = \sqrt{M^2c^4 + E_\gamma^2}$$

Squaring both sides,

$$\left(m_n + m_p\right)^2 c^4 + E_\gamma^2 - 2E_\gamma\left(m_n + m_p\right)c^2 = M^2c^4 + E_\gamma^2$$

$$\Rightarrow 2E_\gamma\left(m_n + m_p\right)c^2 = \left(m_n + m_p\right)^2 c^4 - M^2c^4 \quad \ldots(3)$$

$$2E_\gamma\left(m_n + m_p\right)c^2 = \left[B + Mc^2\right]^2 - M^2c^4 \Rightarrow 2E_\gamma Mc^2 = B^2 + 2BMc^2$$

$$E_\gamma - B = \frac{B^2}{2Mc^2}$$

Correct option is (d)

39. A neutron and a proton can undergo radioactive capture at rest : $p + n \rightarrow d + \gamma$. Find the energy of the photon emitted in this capture.

(a) 2.326 MeV (b) 2.234 MeV (c) 2.456 MeV (d) 1.826 MeV

Soln. The energy released in the radioactive capture is

$$Q = \left[m_p + m_n - m_d \right] c^2 = 1.00783 + 1.00867 - 2.01410 \text{ amu} = 2.234 \text{ MeV}.$$

This energy appears as the kinetic energy of the photon and recoil deuteron. Let their respective momenta be p and $-p$. Then,

$$Q = pc + \frac{p^2}{2m_d}, \text{ or } (pc)^2 + 2m_d c^2 (pc) - 2m_d c^2 Q = 0$$

Solving for pc we have $pc = m_d c^2 \left(-1 + \sqrt{1 + \frac{2Q}{m_d c^2}} \right)$

As $Q/m_d c^2 \ll 1$, we can take the approximation

$$p \approx m_d c \left(-1 + 1 + \frac{Q}{m_d c^2} \right) \approx \frac{Q}{c}$$

$$pc \approx Q \Rightarrow E_\gamma = 2.234 \text{ MeV}.$$

Thus the kinetic energy of the recoiling deuteron is

$$E_{\text{recoil}} = \frac{p^2}{2m_d} = \frac{Q^2}{2m_d c^2} = \frac{2.234^2}{2 \times 2.0141 \times 931} = 1.33 \times 10^{-3} \text{ MeV}.$$

Since, $\frac{\Delta E_{\text{recoil}}}{E_\gamma} = \frac{1.34 \times 10^{-3}}{2.234} = 6.0 \times 10^{-4}$,

the recoiling of the deuteron does not significantly affect the energy of the emitted photon, its effect being of the order 10^{-4}.

Correct option is (b)

CAREER ENDEAVOUR

PRACTICE SET

1. An atomic bomb consisting of ^{235}U explodes and release an energy of 10^{14} J. It is known that each ^{235}U which undergoes fusion release 3 neutrons and about 200 MeV of energy. Further, only 20% of the ^{235}U atoms in the bomb undergo fission. The total number of neutrons released is about:
(a) 4.7×10^{24} (b) 9.7×10^{24} (c) 1.9×10^{25} (d) 3.7×10^{25}

2. The condition for spontaneous fission of a nucleus is :
(a) $\frac{Z^2}{A} < 50$ (b) $\frac{Z^2}{A} > 45$ (c) $\frac{Z}{A} > 45$ (d) $\frac{Z}{A} < 45$

3. Muons are produced through the annihilation of particle a and its anti-particle $\bar{a}$, namely the process $a + \bar{a} \rightarrow \mu^+ + \mu^-$. A muon has a rest mass of 105 MeV/c^2 and its proper life time is 2 μs. If the center of mass energy of the collision is 2.1 GeV in the laboratory frame that coincides with the center-of-mass frame, then the fraction of muons that will decay before they reach a detector placed 6km away from the interaction point is
(a) e^{-1} (b) $1 - e^{-1}$ (c) $1 - e^{-2}$ (d) e^{-10}

4. A spin-1/2 particle A undergoes the decay $A \rightarrow B + C + D$ where it is known that B and C are also spin-1/2 particles. The complete set of allowed values of the spin of the particle D is
(a) $\frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, 3, \ldots\ldots$ (b) 0; 1 (c) $\frac{1}{2}$ only (d) $\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}, \ldots$

ANSWER KEY

Questions	1	2	3	4
Option	(b)	(b)	(d)	(c)

Chapter 6

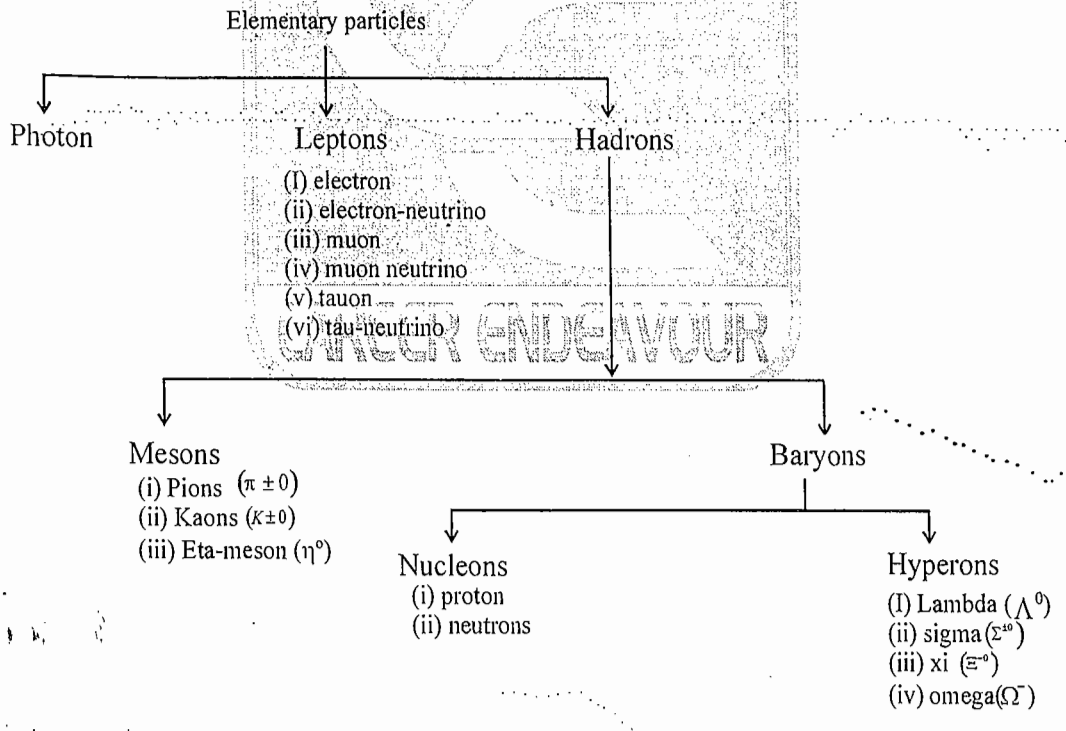
Elementary Particles

Introduction :

It is a dream for physicists to be able to explain the material world, the physical universe entirely. This has driven them continuously to explore the fundamental entities of matter was atom then nucleus, neutron and protons and now the quarks. In this present chapter we will read about all elementary particle their spin charge, isospin, mass etc.

Classification of elementary particles:

The elementary particles may be classified in a number of different ways depending on their masses, interaction, statistics etc. Commonly, however, they are classified into such categories as



Some elementary particle and their properties:

Category	Particles Name	Symbols	Charge	Spin	B	L	s	I_3	Y	Quark structure
	Electron	e^-	–1	½	0	1	0		0	
	Electron-neutrino	ν_e	0	½	0	1	0		0	
Leptons	Muon	μ^-	–1	½	0	1	0		0	
	Muon-Neutrino	ν_μ	0	½	0	1	0		0	
	Tauon	τ	–1	½	0	1	0		0	
	Tauon-Neutrino	ν_τ	0	½	0	1	0		0	
		π^+	+1	0	0	0	0	+1	0	$u\bar{d}$
	Pions	π^0	0	0	0	0	0	0	0	$\frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$
		π^-	–1	0	0	0	0	–1	0	$d\bar{u}$
Mesons		K^+	+1	0	0	0	+1	-½	+1	$u\bar{s}$
	Kaons	K^0	0	0	0	0	+1	-½	+1	$d\bar{s}$
		K^-	–1	0	0	0	–1	½	–1	$s\bar{u}$
	Eta	η^0	0	0	0	0	0	0	0	$\frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s})$
	η^1	η'	0	0	0	0	0	0	0	$(u\bar{u} + d\bar{d} + s\bar{s})\frac{1}{\sqrt{3}}$
Nucleons	Proton	P	+1	½	1	0	0	½	1	uud
	Neutron	n	0	½	1	0	0	-½	1	udd
	Lambda	Λ^0	0	½	1	0	–1	½	0	uds
	Sigma	Σ^+	+1	½	1	0	–1	+1	0	uus
		Σ^0	0	½	1	0	–1	0	0	uds
Hyperons		Σ^-	–1	½	1	0	–1	0	0	dds
	Xi	Ξ^0	0	½	1	0	-2	½	–1	uss
		Ξ^-	–1	½	1	0	-2	-½	-1	dss
	Omega	Ω^-	-1	½	1	0	–3	0	–2	sss

Conservation laws in elementary particle reactions:

- **Exact conservation laws:**
  (i) Conservation of linear momentum (ii) Conservation of angular momentum
  (iii) Conservation of charge (iv) Conservation of baryon number
  (v) Conservation of lepton number
- **Approximate conservation laws:**
  (i) Isospin or isotopic spin(I):
  It arose however from the idea that pairs of particles like nucleons and triplets like pions hardly differ in their mass and may be considered as isotopes and that their charges, differing from each other by unity, suggest space quantization similar to electron spin and orbit in a magnetic field.
- **A multiplet number** (M) is defined as the number of their different charge states. For instance, for nucleons i.e. protons or neutrons - the multiplet number $M = 2$. Similarly, for the triplet of pions, $M = 3$; $M = 2$ for two kaons and two antikaons; $M = 1$ for one neutral η^0.

Now, $M = 2I + 1 \quad \Rightarrow I = (M-1)/2$

- Isospin is treated as a vector $\vec{I}$ of magnitude $\sqrt{I(I+1)}$, like angular momentum, but I is dimensionless. Its component along Z-axis, is given by I_3 which have the allowed values.

  $I, (I-1), (I-2), \ldots\ldots\ldots\ldots -I$

- For nucleons $(M=2)$, $I = (M-1)/2 = \frac{1}{2}$ and the values of I_3 are $+\frac{1}{2}$ and $-\frac{1}{2}$; $I_3 = +\frac{1}{2}$ is assigned to proton, $I_3 = -\frac{1}{2}$ to neutron.

- For pions $(M=3)$, $I = (M-1)/2 = 1$. Hence, $I_3 = +1, 0, -1$. $I_3 = +1$ is assigned to π^+, $I_3 = 0$ to π^0 and $I_3 = -1$ to π^-.

- Isospin is conserved in strong interactions but is violated in electromagnetic and weak interaction. The Z-component of isospin, I_3- is conserved in strong and electromagnetic interaction and not in weak interaction.

(ii) Hypercharge(Y): It is defined as double of the average charge $\bar{Q}$ of the multiplet.

$$Y = 2\bar{Q} \Rightarrow \bar{Q} = Y/2 \quad \Rightarrow \boxed{Q = I_3 + Y/2}$$

- For any strong and electromagnetic interaction, the hypercharge is conserved, i.e. remains invariant. But it need not be conserved in weak interaction.

Example: $p + p \rightarrow \Lambda^0 + K^0 + p + \pi^+$

Hypercharge: $1+1 = 0+1+1+0 \quad \Rightarrow \Delta Y = 0$

(iii) Strangeness number (S): It is defined as the difference of the hypercharge Y and the baryon number B.

$$S = Y - B \quad \Rightarrow Y = S + B$$

i.e., the hyperchange is the sum of the baryon number and the strangeness number

Therefore, $Q = I_3 + \frac{B+S}{2}$

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- Strangeness number S is conserved in strong and electromagnetic reactions. For a weak interactions, $\Delta S = 0$ or ± 1

(iv) Parity:

When particle like neutrinos are emitted during radioactive decay, they show a preferred spin direction. If a neutrino spins in the direction at which a right-handed screw advances, it is said to posses a helicity +1; if however the spin is in the direction of a left-handed screw, the helicity is −1. As the parity P is related to the spin J, the two quantum numbers are usually combined and is symbolised by J^P. So, $\left(\frac{1}{2}\right)^+$ means the J-value is $\frac{1}{2}$ and $P = +1$; $\left(\frac{1}{2}\right)^-$ indicates $J = \frac{1}{2}, P = -1$.

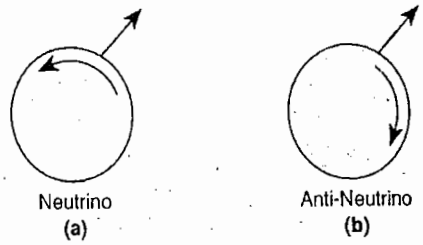


Figure: Helicity of neutrino and antineutrino

- In strong and electromagnetic interactions, parity is conserved but it is not conserved in weak interaction.

(v) Charge conjugation:

Charge conjugation means reversal of the signs of all types of charge i.e. electric, baryonic and laptonic of the particles (Figure below). If a physical law holding for particles also holds for corresponding antiparticles, the principle of charge conjugation is said to be valid.

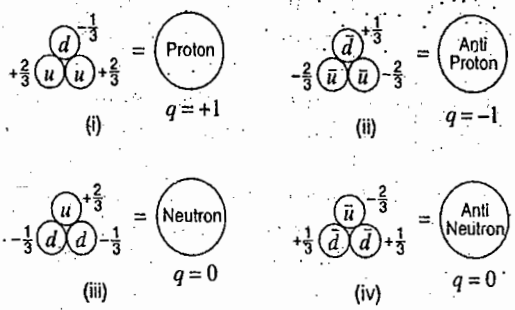


Figure: Formation of proton, antiproton and neutron, antineutron by charge conjugation

- Strong and electromagnetic interactions are charge conjugate invariant. But the weak interaction like β-decay does not obey charge conjugation.

(vi) Time reversal:

The operation T i.e., time-reversal means replacing the time t by $-t$ in all equations of motion i.e., reflection of time axis at the origin of time coordinate in relativistic space-time continum. It is thus, like the parity operation, a discrete change.

- T-operation consists in reversing the signs of momenta $\left(\vec{p} = d\vec{r}/dt\right)$ and angular momenta $\left(\vec{L} = \vec{r} \times \vec{p}\right)$.

  T also transforms the wave function to its complex conjugate. If T is conserved i.e., time-reversal invariance occurs, then the reversed equation of motion is also a valid equation of motion of the system concerned. All the known fundamental equations of motion are invariant in time-reversal.
- Strong and electromagnetic interactions are invariant under time-reversal transformation.

***CPT* theorem:**

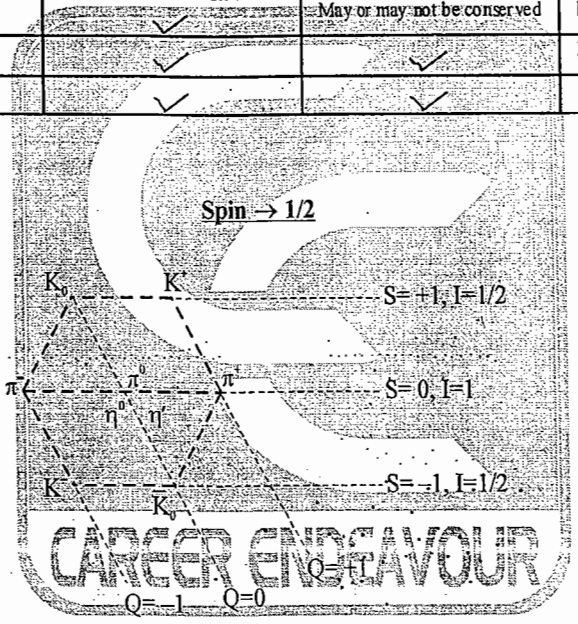
This is an exact conservation law. It states that all interactions in nature are invariant under joint operations of charge conjugation (C), inversion of space coordinates at origin, i.e. parity (P) and reversal of time (T). The order of operations is immaterial.

The invariance of *CPT* transformation implies that if any interaction is not invariant under any one of C, P and T operations, its effect gets compensated by the joint effect of the other two.

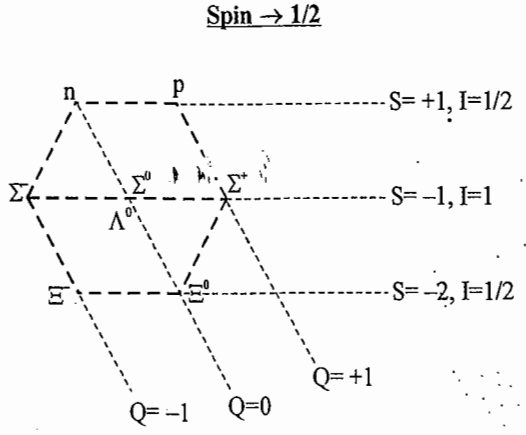
Conserved quantities in different type of reactions:

Conserved Quantities	Strong	Electromagnetic	Weak
1. Charge	✓	✓	✓
2. Linear Mom.	✓	✓	✓
3. Relativistic En	✓	✓	✓
4. Spin	✓	✓	✓
5. L_e	✓	✓	✓
6. L_u	✓	✓	✓
7. L_τ	✓	✓	✓
8. Baryon No. B	✓	✓	✓
9. I_3	✓	✓	×
10. I	✓	May or may not be conserved	×
11. S	✓	✓	May or may not be conserved
12. Y = B+S	✓	✓	May or may not be conserved
13. Parity (P)	✓	✓	×
14. Charge conjugation	✓	✓	×
15. Time reversion (T)	✓	May or may not be conserved	May or may not be conserved
16. CP	✓	✓	May or may not be conserved
17. CPT	✓	✓	✓

Meson Octate:



Baryon Octate:



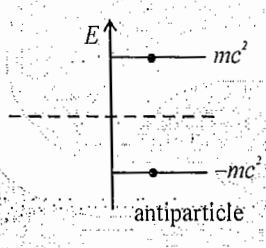
Relationship between particles and antiparticles:

(a) Mass	→	Same
(b) Spin	→	Same
(c) Charge	→	Same but opposite in sign.
(d) Mag. moment	→	Same in magnitude but opposite in sign.
(e) Mean life time of free decay	→	Same
(f) Annihilation	→	In pairs
(g) Creation	→	In pairs
(h) Total isotopic spin	→	same
(i) Third component of isospin(I_3)	→	Same in mag. but opposite in sign.
(j) Intrinsic parity	→	Same for bosons and opposite for Fermions.
(k) Strangeness number (S)	→	Same in mag. but opposite in sign.
(l) Lepton number (L)	→	Same in mag. but opposite in sign.
(m) Baryon number (B)	→	Same in mag. but opposite in sign.
(n) Hypercharge (Y)	→	Same in mag. but opposite in sign.

The particle and antiparticle have same symbol except a bar over particle for antiparticle.

Concept of antiparticle by Dirac:

$$E = \pm\sqrt{p^2c^2 + m_0^2c^4}$$



The negative solution of Schrodinger equation is also possible. But Dirac said that all the negative energy states are already filled. If electron given enough energy it can come out the filled negative state to positive energy.

The vaccant state in negative energy behaves like holes → antiparticles

Quark model:

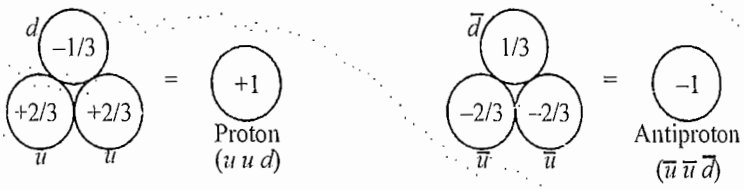
The elementary particles can be conceived (as far as isospin and hyper charge are concerned) as being built out of combination of quarks. The original three quarks were called up (u), down (d) and strange (s).

Each quarks has an anti-quark associated with it $(\overline{u}, \overline{d}, \overline{s})$. Since a quark and its anti-quark have opposite quantum numbers. They can be created from energy. In the reverse process, a quark and its anti-quark annihilate and give energy.

Thus, energy → $u + \overline{u}$

$d + \overline{d}$ → energy

Quark models for proton and antiproton.



Coloured quark :

Quarks and antiquarks have an additional property of some kind that can be manifested in a total of six different ways, rather as electric charge is a property that can be manifested in the two different ways that have come to be called positive and negative. In the case of quarks, this property became known as colour and its three possibilities were called red, green and blue. The antiquark colours are antired, antigreen and antiblue.

According to the colour hypothesis, all three quarks in a baryon have different colours which satisfies the exclusion principle since all are then a different states even if two or three are otherwise identical

The rules for combining colours are the following:

- A colour and its anticolour cancel out. This is called colourless or white.
- All three colours or all three anticolours in combination cancel out and give colourless
- All hadrons are colourless.

Mesons consist of a quark-antiquark pair of a particular colour and its anticolour. Baryons are made up of three quarks, one of each colour. Thus mesons and baryons are white or colourless.

Charm, Bottom and Top quarks:

Besides the three quarks (u, d, s) three more quarks are suggested in order to have a significant analysis of the symmetries. These are charm (c), top (t) and bottom (b).

Thus over all there are six quarks. The various characteristics of these quarks are given in the following table:

Quark	I	I_3	B	S	Y	Q	*spin*	C	E	B	T	*Mass* (GeV/c^2)
Up(u)	1/2	+1/2	1/3	0	1/3	2/3	½	0	0	0	0	0.39
Down(d)	1/2	−1/2	1/3	0	1/3	−1/3	½	0	0	0	0	0.39
Strange(s)	0	0	1/3	−1	−2/3	−1/3	½	0	0	0	0	0.51
Charm(c)	0	0	1/3	0	1/3	2/3	½	1	1	0	0	1.2
Top(t)	0	0	1/3	0	1/3	−1/3	½	0	0	0	1	174
Bottom(b)	0	0	1/3	0	1/3	2/3	½	0	0	1	0	4.2

Few examples of particle quark structure:

$K_0 = \bar{s}d;\ K^+ = \bar{s}u;\ \pi^- = d\bar{u}$, π^0 and η^0 may be $u\bar{u}$ and $d\bar{d}$

$$\pi^0 = \frac{1}{\sqrt{2}}\left(u\bar{u} - d\bar{d}\right)$$

$$\eta' = \frac{1}{\sqrt{3}}\left(u\bar{u} + d\bar{d} + s\bar{s}\right)$$

$\pi^+ = u\bar{d};\ K^- = s\bar{u};\ \bar{K}_0 = s\bar{d}$

$n = udd;\ p = uud$

$\Sigma^- = sdd;\ \Sigma^+ = suu;\ \Xi^- = ssd;\ \Xi^0 = ssu;\ \Omega^- = sss$

λ^0 and Σ^0 may both be sud

$$\Sigma^0 = \frac{1}{\sqrt{2}}\left(sud + sud\right) \longrightarrow \text{parity} = +1$$

$$\lambda^0 = \frac{1}{\sqrt{2}}\left(sud - sud\right) \longrightarrow \text{parity} = -1$$

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Baryon Resonances :

Particle	I	I_3	B	S	Y	Structure
Δ^{++}	$3/2$	$3/2$	1	0	1	uuu
Δ^{+}	$3/2$	$1/2$	1	0	1	uud
Δ^{0}	$3/2$	$-1/2$	1	0	1	udd
Δ^{-}	$3/2$	$-3/2$	1	-1	0	ddd
Σ^{*+}	1	1	1	-1	0	uus
Σ^{*0}	1	0	1	-1	0	uds
Σ^{*-1}	1	-1	1	-1	0	dds
Ξ^{*0}	$1/2$	$1/2$	1	-2	-1	uss
Ξ^{*-}	$1/2$	$-1/2$	1	-2	-1	dss
Ω^{-}	0	0	1	-3	-2	sss

The four fundamental forces:

In nature, there are four different types of fundamental interactions. They are:

1. The gravitational force :

The gravitational force is the oldest known force. It holds us to the surface of the earth and holds the entire universe together. It is a long-range force varying as $\frac{1}{r^2}$. It has life time, $\tau = 10^{16}$ sec.

2. The electromagnetic force:

The electromagnetic force was the second force known. In fact it was originally two forces the electric force and magnetic force. The electromagnetic force hold the atoms, molecules, solid and liquid together. It is also a long rang force varying as $\frac{1}{r^2}$. It has a life time, $\tau = 10^{-20}$ sec.

3. Weak interaction force:

This fundamental interaction involves leptons and hedrons. The β-decay of radioactive nuclei and decays of strong particles are typically of weak interaction. It has a life time, $\tau = 10^{-10}$ sec.

4. Strong interaction force :

The strong nuclear force is responsible for holding the nucleons together. It is the strongest of all the forces but is a very short range force. Its effect occur within a distance of about 10^{-15} m. The strong nuclear force acts only on the hadrons. It has a life time, $\tau = 10^{-23}$ sec.

Kinematics of high energy collisions (Relativistics):

Many elementary particles are the product of high energy collision. We shall now study the kinematics of such collision, restricting to those result in the production of two product particles only. But it can be easily generalised. For compactness, we shall use a system of unit where the light velocity (c) would be taken as unity, i.e. $c = 1$, so that mc^2 is denoted by the mass m, the energy pc, momentum p etc.

Let us take reaction

$$A + B \rightarrow C + D \quad \ldots (1)$$

The conservation principles of energy and momentum

$$E_A + E_B = E_C + E_D \quad \ldots (2)$$

And $$P_A + P_B = P_C + P_D \quad \ldots (3)$$

where, E's represent total energies and P's the momenta of the individual particles.

Using relativistic relations for different particles, we get

$$E_n^2 = P_n^2 + m_n^2, \; n = 1, 2, 3, \ldots\ldots \quad \ldots (4)$$

From relativity again the total energy E and the components of momentum vector $\vec{P}$ constitute a four vector $P_\mu = (E; P)$ and these scalar product $P_\mu P_\mu$ in 4-space is an invariant at a particular time.

$$\therefore \quad P_\mu P_\mu = E^2 - P^2 = \text{invariant} \quad \ldots (5)$$

P^2 being the square of 3d-momentum vector.

Therefore, for a system of particles, in initial or final state.

$$P_\mu P_\mu = (\Sigma E_n)^2 - (\Sigma P_n)^2 = \text{invariant} \quad \ldots . (6)$$

In reference to the reaction (1), therefore, we have

$$(P_\mu P_\mu)_{initial} = (E_A + E_B)^2 - (P_A + P_B)^2$$

$$= \left(\sqrt{P_A^2 + m_A^2} + \sqrt{P_B^2 + m_B^2}\right)^2 - (P_A + P_B)^2 \quad \ldots (7)$$

In Lab-frame, assuming the target B to be at rest $P_B = 0$. The kinetic energy T_A of the incident particle A is then

$$T_A = E_A - m_A = \sqrt{P_A^2 + m_A^2} - m_A$$

Therefore, from (8), we obtain the relation,

$$(P_\mu P_\mu)_{\text{initial}} = \left(\sqrt{P_A^2 + m_A^2} + m_B\right)^2 - P_A^2$$

$$= m_A^2 + m_B^2 + 2m_B\sqrt{P_A^2 + m_A^2}$$

$$= m_A^2 + m_B^2 + 2m_B(m_A + T_A)$$

$$= (m_A + m_B)^2 + 2m_A T_A \quad \ldots (8)$$

Threshold energy:

At the threshold, A_C and A_D are produced with zero momentum, i.e. we then have $P_C = P_D = 0$ using equation (8)

$$(P_\mu P_\mu)_{final} = (m_C + m_D)^2 \quad \ldots (9)$$

Substituting $m_i = m_A + m_B$ and $m_f = m_C + m_D$ and exploting the invariance property we get from equations (8) and (9)

$$m_i^2 + 2m_B T_A = m_f^2$$

$$\Rightarrow \quad \boxed{T_A = (m_f^2 - m_i^2)/2m_B} \quad \ldots (10)$$

Again, Q-value of the reaction is given by

$$Q = m_A + m_B - m_C - m_D = m_i - m_f \quad \ldots (11)$$

Threshold energy,

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$$E_{th} = T_A = \frac{(m_f + m_i)(m_f - m_i)}{2m_B} \text{ (using (10))}$$

$$= -\frac{Q}{2m_B}(m_i + m_f) = \frac{Q}{2m_B}(Q - 2m_i)$$

$$\boxed{E_{th} = Q\left[\frac{Q}{2m_B} - \frac{m_A + m_B}{m_B}\right]} \quad \ldots (12)$$

1. **Non-relativistic collision:** Hence, $Q << m_A$ (or m_B) so, from equation (12) We obtain

$$\boxed{E_{th} = -Q\left(\frac{m_A + m_B}{m_B}\right)} \quad \ldots (13)$$

2. **Extreme relativistic case :** Here $Q >> m_A$ and also m_B so that

$$\boxed{E_{th} = \frac{Q^2}{2m_B}} \quad \ldots (14)$$

SOLVED PROBLEMS

1. If in the following reaction, the incident kaon has a kinetic energy of 1.63 GeV, calculate the total energy to be divided between the four recoiling particles.

$$K^- + p^+ \rightarrow \Sigma^- + \pi^+ + \pi^- + \pi^+$$

The mass-energy of π-mesons are 139.6 MeV, Σ^- = 1197.3 MeV, proton = 938.3 MeV and $\bar{K}^-$ = 493.8 MeV

Soln. Energy of particles (rest mass energy + kinetic energy) taken together are:

$\bar{K}^- = 0.4938$ GeV $\left(1\text{ GeV} = 10^3\text{ MeV}\right)$
$p^+ = 0.9383$ GeV
$E_{\bar{K}^-} = 1.63$ GeV

$\Rightarrow$ Total $E = 3.0621$ GeV

Energy of the four recoiling particles are:

$\Sigma^- = 1.1973$ GeV
$\pi^+ = 0.1396$ GeV
$\pi^- = 0.1396$ GeV
$\pi^+ = 0.1396$ GeV

Total energy $= 1.6161$ GeV

Therefore, excess energy = 3.0621 –1.6161 = 1.446 GeV
Therefore, average energy per particle = 1.446/4 = 0.3615 GeV = 361.5 MeV

2. If a pion decays from rest to give a muon of 4.0 MeV energy, what is the kinetic energy of the accompanying neutrino? What is the mass of the neutrino in the process?

Soln. The decay mode of pion is given by, $\pi^- \to \mu^+ + \nu_\mu + E$

Therefore, energy, $E = (m_\pi - m_\mu) \times c^2$ (neutrino has zero 'rest' mass)

$$= (273m_e - 207m_e) \times c^2 = 66m_e \times c^2 = 66 \times 0.51 \text{ MeV} = 33.7 \text{ MeV}$$

Since muon takes 4.0 MeV of energy, the kinetic energy of the accompanying neutrino is

$$(33.7 - 4.0) \text{ MeV} = 29.7 \text{ MeV}$$

Therefore, mass of neturino = $(29.7/0.51)m_e = 58.23\ m_e$.

3. Find the value of third component of isotopic spin of Ξ^- in the following strong interaction:

$$\pi^+ + n \to \Xi^- + K^+ + K^+$$

Soln. The interaction is $\pi^+ + n \to \Xi^- + K^+ + K^+$

$$I: 1 + \frac{1}{2} \to \frac{1}{2} + \frac{1}{2} + \frac{1}{2}$$

$$I_3: +1 - \frac{1}{2} \to I_3 + \frac{1}{2} + \frac{1}{2}$$

Therefore, I_3 for $\Xi^- = -\frac{1}{2}$

4. Identify the type of the following interaction from conservation laws:

$$\Sigma^0 \to \Lambda^0 + \gamma \quad (\text{life time} \le 10^{-14}\text{ s})$$

Soln. We have $\Sigma^0 \to \Lambda^0 + \gamma$

Charge, Q	:	$0 \to 0 + 0$	$\Rightarrow \Delta Q = 0$
Baryon number, B	:	$+1 \to +1 + 0$	$\Rightarrow \Delta B = 0$
Lepton number, L	:	$0 \to 0 + 0$	$\Rightarrow \Delta L = 0$
Strangeness no. S	:	$-1 \to -1 + 0$	$\Rightarrow \Delta S = 0$
Isospin I_3	:	$0 \to 0 + 0$	$\Rightarrow \Delta I_3 = 0$
Isospin I	:	$1 \to 0 + 0$	$\Rightarrow \Delta I \neq 0$

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Its half-life is $\le 10^{-14}$ s, I_3 is conserved and isospin (I) is not conserved which points to the fact that it cannot be a strong interaction. So, it is an electromagnetic interaction and a γ-photon is produced.

5. Identify the unknown particle in the reactions given below, using the conservation laws.

(i) $\mu^- + p \to {}^1_0n + ...$ (ii) $\pi^- + p \to K^0 + ...$

Soln. (i) The given reaction is : $\pi^- + p \to {}^1_0n + ...$

The unknown particle must be zero charge and mass, spin $\frac{1}{2}$ and lepton number 1 as they are conserved.

Since the interacting particle is π^- meson, the unknown particle is identified as mu-neutrino, ν_μ

(ii) The reaction is $\pi^- + p \to K^0 + ...$

For charge conservation, Q	:	$-1 + 1 \to 0 + Q$	$\Rightarrow Q = 0$
Conservation of baryon no. B	:	$0 + 1 \to 0 + Q$	$\Rightarrow B = +1$
Strangeness conservation, S	:	$0 + 0 \to -1 + S$	$\Rightarrow S = +1$
Third component of isospin, I_3	:	$-1 + \frac{1}{2} \to -\frac{1}{2} + I_3$	$\Rightarrow I_3 = 0$

Therefore, the unknown particle has charge zero, baryon number +1, strangeness number +1 and third component of isospin 0. So the particle could be Λ^0 or Σ^0.

6. Check if the following reactions are allowed or forbidden.

(i) $\pi^- + p \to \Lambda^0 + \pi^0$ (ii) $p + p^- \to 2\pi^+ + 2\pi^- + 2\pi^0$

Soln. (i) The reaction is : $\pi^- + p \to \Lambda^0 + \pi^0$

Q: $-1+1 \to 0+0$ $\Rightarrow \Delta Q = 0$ $Q \to$ conserved

B: $0+1 \to +1+0$ $\Rightarrow \Delta B = 0$ $B \to$ conserved

S: $0+1 \to -1+0$ $\Rightarrow \Delta S \neq 0$ $S \to$ not conserved

It is a strong interaction where the charge and baryon number are conserved. But since the strangeness number is not conserved, the reaction is forbidden.

(ii) The reaction is : $p + p^- \to 2\pi^+ + 2\pi^- + 2\pi^0$

Q: $+1-1 \to 2-2+0;$ $\Delta Q = 0$

B: $+1-1 \to 0+0+0;$ $\Delta B = 0$

S: $0+0 \to 0+0+0;$ $\Delta S = 0$

So, the above reaction is an allowed reaction.

7. An ultra-relativistic proton moves in a magnetic field. Can it radiate π^+, π^- and π^0, electrons and positrons?

Soln. If the energy of the proton is sufficiently large, it can radiate π^0 and π^+ mesons, and also positrons. The reactions are:

$$p \to p + \pi^0; \quad p \to n + \pi^+; \quad p \to n + e^+ + \nu$$

But π^- mesons and electrons cannot be radiated.

8. Allocate the Isospin to the strange particles from following spins.

(a) $\pi^- + p \to \pi^o + K^o$ (b) $p + p \to \pi^o + K^+ + p$

(c) $\pi^+ + n \to \pi^o + K^+$ (d) $\pi^- + p \to \Sigma^- + K^+$

(e) $\pi^+ + p \to \Sigma^+ + K^+$ (f) $\pi^+ + n \to \Xi^- + K^+ + K^+$

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Soln. (a) $\pi^- + p \to \pi^o + K^0$

I_3 -1 $\frac{1}{2}$ 0 $-\frac{1}{2}$

(b) $p + p \to \pi^0 + K^+ + p$

I_3 $\frac{1}{2}$ $\frac{1}{2}$ 0 $\frac{1}{2}$ $\frac{1}{2}$

(c) $\pi^+ + n \to \pi^0 + K^+$

I_3 $+1$ $-\frac{1}{2}$ 0 $\frac{1}{2}$

(d) $\pi^- + p \to \Sigma^- + K^+$

I_3 -1 $\frac{1}{2}$ -1 $\frac{1}{2}$

(e) $\pi^+ + p \to \Sigma^+ + K^+$

I_3 1 $\frac{1}{2}$ $+1$ $\frac{1}{2}$

(f) $\pi^+ + n \to \Xi^- + K^+ + K^+$

I_3 1 $-\frac{1}{2}$ $-\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$

9. Which of the following reactions are allowed and forbidden under the conservation of strangeness, conservation of baryon number and conservation of charge.

$\pi^+ + n \to \Lambda^o + K^+$

$\pi^+ + n \to K^o + K^+$

$\pi^+ + n \to \overline{K}^o + \Sigma^+$

$\pi^+ + n \to \pi^- + p$

$\pi^- + p \to \Lambda^o + K^o$

$\pi^- + p \to \pi^o + \Lambda^o$

Soln. **(i)**

	$\pi^+ + n \to \Lambda^0 + K^+$				
Q	1	0	0	1	$\Delta Q = 0$
B	0	1	1	0	$\Delta B = 0$
S	0	0	−1	1	$\Delta S = 0$

Allowed

(ii)

	$\pi^+ + n \to K^o + K^+$				
Q	1	0	0	1	$\Delta Q = 0$
B	0	1	0	0	$\Delta B \neq 0$
S	0	0	1	1	$\Delta S \neq 0$

Not allowed

(iii)

	$\pi^+ + n \to \bar{K}^0 + \Sigma^+$				
Q	1	0	0	1	$\Delta Q = 0$
B	0	1	0	1	$\Delta B = 0$
S	0	0	−1	−1	$\Delta S = 2$

Not allowed

(iv)

	$\pi^+ + n \to \pi^- + p$				
Q	1	0	−1	1	$\Delta Q \neq 0$
B	0	1	0	1	$\Delta B = 0$
S	0	0	0	0	$\Delta S = 0$

Not allowed.

(v)

	$\pi^- + p \to \pi^0 + \Lambda^0$				
Q	−1	1	0	0	$\Delta Q = 0$
B	0	1	0	1	$\Delta B = 0$
S	0	0	0	1	$\Delta S \neq 0$

Not allowed.

(vi)

	$\pi^- + p \to \pi^0 + \Lambda^0$				
Q	−1	1	0	0	$\Delta Q = 0$
B	0	1	0	1	$\Delta B = 0$
S	0	0	0	−1	$\Delta S \neq 0$

Not allowed.

10. Calculate the energy of the neutron produced when a slow negative pion is captured by a proton. Should neutron be treated relativistically?

Soln: $\pi^- + p^+ \to n^0 + \gamma + Q$

$$140 + 938 = \sqrt{939^2 + E_\gamma^2} + E_\gamma$$

$$\Rightarrow \quad 1078^2 + E_\gamma^2 - 2\times 1078 E_\gamma = 939^2 + E_\gamma^2$$

$$\Rightarrow \quad E_\gamma = \frac{1078^2 - 939^2}{2\times 1078} = 130 \text{ MeV}$$

So, $E_n = 1078 - 939 - 130 = 9$ MeV

So, kinetic energy of neutron is 9 MeV. It is very small as compared to rest mass energy of neutron, hence neutron should not be treated relativisiticaly.

11. Classify the following processes in terms of the type of interaction.

Soln: $\pi^- + p \to \Lambda^0 + K^0$; $\pi^- + p \to \pi^0 + n$; $p + \gamma \to p + \pi^0$; $\Sigma^0 \to \Lambda^0 + \gamma$; $\pi^0 \to \gamma + \gamma$; $K^0 \to \pi^+ + \pi^-$; $\Lambda^0 \to p + \pi^-$; $\Xi^- \to \Lambda^0 + \pi^-$; and $\Lambda^0 \to p + e^- + \bar{\nu}$.

In the reactions $\pi^- + p \to \Lambda^0 + K^0$ and $\pi^- + p \to \pi^0 + n$, the interaction is a short-range force between Hadrons $(\pi, p, n, \Lambda \text{ and } K)$ corresponding to one pion-exchange. These reactions obey selection rules

$\Delta B = 0, \Delta Q = 0, \Delta Y = 0, \Delta \pi = 0, \Delta T = 0, \Delta T_3 = 0$

hence the interaction is strong.

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In the reactions $p+\gamma \to \pi^0 + p$, $\Sigma^0 \to \Lambda^0 + \gamma$ and $\pi^0 \to \gamma + \gamma$, the interaction is electromagnetic, as the electromagnetic force arises due to the mechanical effect of the emission and absorption of virtual photons. These reactions obey all above mentioned selection rules except the rule for conservation of isospin T, hence, the interaction in these processes is electromagnetic.

In the reactions $K^0 \to \pi^+ + \pi^-$, $\Lambda^0 \to p + \pi^-$, $\Xi^- \to \Lambda^0 + \pi^-$ and $\Lambda^0 \to p + e^- + \bar{\nu}$, the interaction is weak, as the weak interaction is due to the leptonic decay of strange and non-strange particles and due to non-leptonic decay of strange particles. These reactions do not conserve parity, strangeness, isospin and the third component of isospin, hence the interaction in these processes is weak.

12. What are the possible values of isotopic spin for the following systems? (a) $A\pi^+$ meson and an antiproton, (b) two neutrons, (c) $a\pi^+$ meson and $a\Lambda^c$, (d) $a\pi^+$ and $a\pi^0$ meson, (e) au and $a\bar{u}$ quark, (f) a c, b and an s quark.

Soln. $I(u) = \frac{1}{2} = I(d)$ while $I(s) = I(c) = I(b) = I(t) = 0$

(a) Using the composition law for isospin, we get

$$I(\pi^+ \bar{p}) = \frac{3}{2}, \frac{1}{2}, \quad I_3(\pi^+ \bar{p}) = I_3(\pi^+) + I_3(\bar{p}) = 1 - \frac{1}{2} = \frac{1}{2}$$

Since $I_3 = \frac{1}{2}$ can be a legitimate isospin projection for both $I = \frac{3}{2}$ and $I = \frac{1}{2}$, we conclude that the composite state $(\pi^+ \bar{p})$ can have either $I = \frac{3}{2}$ or $I = \frac{1}{2}$.

(b) A composite state of two neutrons can have $I(nn) = 1, 0$

$$I_3(nn) = I_3(n) + I_3(n) = \frac{1}{2} - \frac{1}{2} = -1.$$

Since $I_3 = -1$ cannot be an isospin projection of a state with $I = 0$, we conclude that a composite state of two neutrons can only be $I = 1$.

(c) The Λ^0 particle is an isosinglet ($I = 0$) and, consequently, we have

$$I(\pi^+ \Lambda^0) = 1$$

This composite state is therefore $I = 1$.

(d) We can have $I(\pi^+ \pi^0) = 2, 1, 0$

$$I_3(\pi^+ \pi^0) = I_3(\pi^+) + I_3(\pi^0) = 1 + 0 = 1$$

A state with two π mesons can have $I = 2$, $I = 1$ or $I = 0$. However, $I = 0$ has no projection of $I_3 = 1$, and the composite state $(\pi^+ \pi^0)$ must therefore have $I = 2$ or $I = 1$.

(e) For a composite system of a u and a $\bar{u}$ quark, we have

$$I(u\bar{u}) = 1, 0$$

$$I_3(u\bar{u}) = I_3(u) + I_3(\bar{u}) = \frac{1}{2} - \frac{1}{2} = 0$$

Since the projection $I_3 = 0$ is possible for both $I = 1, 0$ states, we conclude that the composite state $(u\bar{u})$ can have $I = 1$ or $I = 0$.

(f) All quarks with flavor quantum numbers are isosinglets. Thus, we have

$$I(bcs) = 0$$

This composite state would therefore be a baryon with unique isospin $I = 0$.

13. A K-meson (with a rest mass of 494 MeV) at rest decays into a muon (with a rest mass of 106 MeV) and a neutrino. The energy of the neutrino, which can be taken as massless, approximately.

(a) 120 MeV (b) 236 MeV (c) 300 MeV (d) 388 MeV **[JEST 2013]**

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Soln. The given reaction, $K \to \mu + \nu$

By conservation of energy

$$E_\nu = \frac{(m_K^2 - m_\mu^2)c^2}{2m_K} = \left[\frac{\left(\frac{494}{c^2}\right)\times\left(\frac{494}{c^2}\right) - \left(\frac{106}{c^2}\right)\left(\frac{106}{c^2}\right)}{\left(2\times\frac{494}{c^2}\right)}\right]$$

$$= \left(\frac{244036 - 11236}{988}\right)\times\frac{c^2\times c^2}{c^2\times c^2} = \frac{232800}{988} = 235.628\ \text{MeV}$$

$\Rightarrow$ $\boxed{E_\nu \cong 236\ \text{MeV}}$

Correct option is (b)

14. The reaction $e^+ + e^- \to \gamma$ is forbidden because, **[JEST 2015]**

(a) lepton number is not conserved (b) linear mementum is not conserved

(c) angular momentum is not conserved (d) charge is not conserved

Soln. For $e^+ + e^- \to \gamma$

Lepton number (L): (i) $(-1) + (1) \to 0,\ \therefore \Delta L = 0$

Charge (Q): (ii) $(+1) + (-1) \to 0,\ \therefore \Delta Q = 0$

Since the rotation of antiparticle and particle pairs are opposite to each other.

Here e^+ and e^- are rotating in opposite directions with equal magnitude. Therefore, there angular momentum $(\vec{L} = \vec{r} \times \vec{p})$ will be conserved in left hand side. But for γ it will be of some magnitude.

Therefore, angular momentum of the given reaction is not conserved.

Correct option is (c)

15. The interaction strength of the recently-discovered Higgs boson (mass approximately 125 GeV/c^2) with any other elementary particle is proportional to the mass of that particle. Which of the following decay processes will have the greatest probability? **[TIFR 2014]**

(a) Higgs boson decaying to a top quark + a top anti-quark

(b) Higgs boson decaying to a bottom quark + a bottom anti-quark

(c) Higgs boson decaying to an electron and a positron

(d) Higgs boson decaying to a neutrino-antineutrino pair.

Soln. Since, Higgs Boson has mass approximately 125 GeV/c^2. The decay of this Higgs boson will be possible in the form of a bottom quark plus a bottom antiquark because bottom quarks are the only quarks that have mass of the order of 20 GeV/c^2. Top quraks have mass 5.5 GeV/c^2 while charm have mass 1.55 GeV/c^2.

Correct option is (b)

16. Consider the following reaction involving elementary particles: **[TIFR 2015]**

(A) $\pi^- + p \to K^- + \Sigma^+$ (B) $K^- + p \to K^- + \rho^+$

Which of the following statements is true for strong interactions ?

(a) (A) and (B) are both forbidden (b) (B) is allowed but (A) is forbidden

(c) (A) is allowed but (B) is forbidden (d) (A) and (B) are both allowed

Soln. Given reaction (A) $\pi^- + p \to K^- + \Sigma^+$ and (B) $K^- + p \to K^- + \rho^+$

These reactions will follow strong interactions, if they will be allowed on the basis of conservation laws ($\because$ all the conservation laws are valid in strong interactions) or in other words we can say all the quantities such as charge, linear momentum, spin parity etc. are conserved in strong interactions.

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Let us check conservation laws one by one:

For (A) $\pi^- + p \to K^- + \Sigma^+$

Charge (Q) : $(-1) + (1) \to (-1+1) \Rightarrow \Delta Q = 0$

Spin (s) : $(0) + \left(\frac{1}{2}\right) \to (0) + \left(\frac{1}{2}\right) \Rightarrow \Delta s = 0$

Lepton number (L) : $(0) + (0) \to (0) + (0) \Rightarrow \Delta L = 0$

Baryon number (B) : $(0) + (1) \to (0) + (1) \Rightarrow \Delta B = 0$

I_3 : $(-1) + \left(\frac{1}{2}\right) \to \left(\frac{1}{2}\right) + (1) \Rightarrow \Delta I_3 \neq 0$

Therefore, reaction A is forbidden.

For (B) $K^- + p \to K^- + \rho^+$

Charge Q : $(-1) + (1) \to (-1) + (1) \Rightarrow \Delta Q = 0$

Spins s : $(0) + \left(\frac{1}{2}\right) \to (0) + (0) \Rightarrow \Delta s \neq 0$

Therefore, reaction B is also forbidden.

Correct option is (a)

17. Consider the hyperon decay (1) $\Lambda \to n + \pi^0$ followed by (2) $\pi^0 \to \gamma\gamma$. If the isospin component, baryon number and strangeness quantum numbers are denoted by I_z, B and S respectively, then which of the following statements is completely correct ? **[TIFR 2016]**

(a) In (1) I_z is not conserved, B is conserved, S is not conserved;
In (2) I_z is conserved, B is conserved, S is conserved.

(b) In (1) I_z is conserved, B is not conserved, S is not conserved
In (2) I_z is conserved, B is conserved, S is conserved

(c) In (1) I_z is not conserved, B is conserved, S is not conserved
In (2) I_z is not conserved, B is conserved, S is conserved

(d) In (1) I_z is not conserved, B is conserved, S is conserved
In (2) I_z is conserved, B is conserved, S is conserved

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Soln. For (i) $\Lambda \to n + \pi^0$

Baryon number (B) $(+1) \to (+1) + 0 \Rightarrow \Delta B = (1) - (1) = 0$

Isospin (I_z) $(+1) \to \left(-\frac{1}{2}\right) + 0 \Rightarrow \Delta I_z \neq 0$

Strangeness (S) $(-1) \to 0 + 0 \Rightarrow \Delta S \neq 0$

Therefore, in (i) I_z is not conserved, B is conserved and S is not conserved.

For (ii) $\pi^0 \to \gamma\gamma$

Baryon number (B) $0 \to 0 + 0 \Rightarrow \Delta B = 0$

Isospin (I_z) $0 \to 0 + 0 \Rightarrow \Delta I_z = 0$

Strangeness (S) $0 \to 0 + 0 \Rightarrow \Delta S = 0$

Therefore, in (ii) I_z is conserved, B is conserved and S is conserved.

Correct option is (a)

18. The quark content of Σ^+, K^-, π^- and p is indicated.

$|\Sigma^+\rangle = |uus\rangle$; $|K^-\rangle = |s\bar{u}\rangle$; $|\pi^-\rangle = |d\bar{u}\rangle$; $|p\rangle = |uud\rangle$

In the process $\pi^- + p \to K^- + \Sigma^+$;

Considering strong interactions only, which of the following is true?

(a) the process is allowed because, $\Delta S = 0$

(b) the process is allowed because, $\Delta I_3 = 0$

(c) the process is not allowed because, $\Delta S \neq 0$ and $\Delta I_3 \neq 0$

(d) the process is not allowed because the baryon number is violated.

Soln. Given nuclear reaction, $\pi^- + p \to K^- + \Sigma^+$

$$\Delta I_3 = \left(-1+\frac{1}{2}\right) - \left(-\frac{1}{2}+1\right) = \left(-\frac{1}{2}\right) - \left(\frac{1}{2}\right) = -1 \neq 0$$

$$\Delta S = (0+0) - (-1-1) = 2 \neq 0; \quad \Delta B = (0+1) - (0+1) = 0$$

Correct option is (c)

19. It is well-known that the energy of the Sun arises from the fusion of hydrogen nuclei (protons) inside the core of the Sun. This takes place through several mechanisms, each resulting in emission of energy. Which of the following reactions is NOT possible during the proton fusion inside the Sun?

(a) ${}_1^1H + {}_1^1H \longrightarrow {}_2^2He$ (b) ${}_1^2H + {}_1^1H \longrightarrow {}_2^3He$

(c) ${}_1^1H + {}_1^1H \longrightarrow {}_1^2H + e^+ + \nu_e$ (d) ${}_1^1H + {}_1^1H + {}_1^1H + {}_1^1H \longrightarrow {}_1^4He + 2e^+$

Soln. Option (b) is correct because two protons have converted to neutron so there will be also 2 neutrino.

According to β^+-decay, $P \to n + e^+ + \nu_e$

Correct option is (b)

20. Let E_N be the energy released when one mole of pure ^{235}U undergoes controlled fission, and E_C be the energy released when one mole of pure carbon undergoes complete combustion. The ratio E_N/E_C will have the order of magnitude **[TIFR 2013]**

(a) 10^4 (b) 10^8 (c) 10^9 (d) 10^6

Soln. E_N the energy released will be order of ~MeV and E_C the energy released will be order of ~ eV.

So, $\frac{E_N}{E_C} \sim \frac{\text{MeV}}{\text{eV}}$; so, $\frac{E_N}{E_C} \sim 10^6$

Correct option is (d)

21. An alpha particle of energy E is shot towards a gold nucleus $\left({}_{79}^{197}Au\right)$. At the distances much larger than the nuclear size R_N, the dominant force is the Coulomb repulsion, but at distances comparable to the nuclear size the dominant force is the strong nuclear attraction. These combine to form a potential barrier of height V_C. If $E < V_C$, the probability that the alpha particle will fuse with the gold nucleus can be written (in terms of a dimensionless positive constant k) as **[TIFR 2014]**

(a) zero (b) $\frac{kE}{\sqrt{k^2E^2 + V_C^2}}$ (c) $k\left(1 - \frac{E}{V_C}\right)$ (d) $\exp\left(-\frac{kV_C}{E}\right)$

Soln. Therefore, transmission probability is proportional to $e^{-\gamma d} \Rightarrow \gamma = \sqrt{\frac{2m}{\hbar^2}(V-E)}$

Therefore, probability will always be exponentially decay and will decrease as V_C will increase as E will increase.

Only option that is fulfilling this criteria is (d)

Correct option is (d)

22. The maximum relativistic kinetic energy of β particles from a radioactive nucleus is equal to the rest mass energy of the particle. A magnetic field is applied perpendicular to the beam of β particles, which bends it to a circle of radius R. The field is given by : **[JEST 2016]**

(a) $3m_0c/eR$ (b) $\sqrt{2}m_0c/eR$ (c) $\sqrt{3}m_0c/eR$ (d) $\sqrt{3}m_0c/2eR$

Soln. Kinetic energy of β- particle $= m_0 c^2$ (given)

Therefore, energy - rest mass energy = kinetic energy

$$mc^2 - m_0c^2 = m_0c^2 \quad \ldots(1)$$

$$mc^2 = 2m_0c^2$$

$$\Rightarrow \frac{m_0c^2}{\sqrt{1-\frac{v^2}{c^2}}} = 2m_0c^2 \Rightarrow \frac{1}{2} = \sqrt{1-\frac{v^2}{c^2}} \Rightarrow \frac{1}{4} = 1-\frac{v^2}{c^2} \Rightarrow \frac{v^2}{c^2} = 1-\frac{1}{4}$$

$$\Rightarrow \frac{v^2}{c^2} = \frac{3}{4} \Rightarrow v = \frac{\sqrt{3}}{2}c \quad \ldots(2)$$

and magnetic field is perpendicular to the beam of β- particle

$$\frac{mv^2}{R} = qvB \Rightarrow B = \frac{mv}{eR}$$

From Eq. (1), $m = 2m_0$ and Eq. (2), $v = \frac{\sqrt{3}}{2}c$

$$B = \frac{2m_0}{eR}\frac{\sqrt{3}}{2}c \Rightarrow B = \frac{\sqrt{3}\,m_0\,c}{eR}$$

Correct option is (d)

23. The mean lifetime of muons in their rest frame is 2.2×10^{-6} s. What is the average distance traveled by muons moving with a speed of $0.6\,c$, [c is the speed of light in vacuum]. **[JNU Ph.D. 2009]**

Soln. Life time in rest frame $= 2.2 \times 10^{-6}$ s

$$\text{Life time in laboratory frame} = \frac{2.2 \times 10^{-6}\,\text{s}}{\sqrt{1-v^2/c^2}}$$

$$v = 0.6\,c$$

$$\text{Life time} = \frac{2.2\ \mu\text{s}}{\sqrt{1-(0.6)^2}}$$

$$\text{Life time} = 2.75\ \mu\text{s}$$

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So, average distance travelled is = (life time) × velocity

$= (2.75 \times 10^{-6}) \times (0.6 \times 3 \times 10^{8})$

$= 4.95 \times 10^{2}$ m

So, average distance traveled = 495 m.

Correct answer is (495 m)

24. A relativistic neutron is travelling at half the speed of light. How much energy is required to increase its speed to 0.6 c ? Compare this with the answer that you would get using non-relativistic (Newtonian) mechanics.

Soln. Initially neutron has velocity = 0.5 c. **[JNU Ph.D. 2010]**

Finally neutron has velocity = 0.6 c.

$$\text{Energy of a relativistic particle} = \frac{m_0 c^2}{\sqrt{1 - v^2/c^2}}$$

So, energy required,

$$= \frac{m_0 c^2}{\sqrt{1-(0.6)^2}} - \frac{m_0 c^2}{\sqrt{1-(0.5)^2}}$$

$$= 1.25\, m_0 c^2 - 1.155\, m_0 c^2$$

$$= 0.095\, m_0 c^2$$

Using Newtonian mechanics,

$$\text{Energy required} = \frac{1}{2} m_0 (0.6c)^2 - \frac{1}{2} m_0 (0.5c)^2 = \frac{1}{2} m_0 c^2 \{0.6^2 - 0.5^2\} = 0.055\, m_0 c^2$$

Therefore, energy required relativistically is more than energy required classically Newtonian mechanics.

$$\text{Therefore, } \frac{\text{Energy required (Relativistically)}}{\text{Energy required (Newtonian)}} = \frac{0.095}{0.055} = 1.73.$$

Correct answer is (1.73)

25. Consider two elementary particles, each of mass m and speed 0.6 c. They undergo a headlong collision and fuse together. Calculate the mass M of the composite particle.

Soln. $m + m \rightarrow M$

Therefore, initial particles both has mass m and velocity 0.6 c.

Therefore, velocity is equal and opposite so momentum of both particles is equal and opposite to each other.

So, momentum initial = 0

Momentum final will be also = 0.

{According to law of conservation of momentum}

So, M has velocity = 0, and also according to law of conservation of energy.

$$\frac{mc^2}{\sqrt{1 - v^2/c^2}} + \frac{mc^2}{\sqrt{1 - v^2/c^2}} = Mc^2$$

Therefore, $v = 0.6c$

$$\frac{m}{\sqrt{1-(0.6)^2}} + \frac{m}{\sqrt{1-(0.6)^2}} = M$$

$$1.25\, m + 1.25\, m = M$$

$$M = 2.50\, m$$

Correct answer is (2.50 m)

CAREER ENDEAVOUR

26. The charmed meson D^0 decays into $K^-\pi^+$. The masses of D, K, $\pi = 1.8, 0.5, 0.15$ GeV/c^2 respectively. What is the momentum of the K-meson in the rest frame of the D^0?
[Take $c = 1$]
(a) 0.82 GeV/c (b) 1.2 GeV/c (c) 0.76 GeV/c (d) 1.45 GeV/c

Soln. In the rest frame of the D^0 meson, momentum conservation gives $\vec{p}_K + \vec{p}_\pi = 0$ **or** $|p_K| = |p_\pi|$.

Energy conservation gives $E_K + E_\pi = m_D$.

i.e., $\sqrt{p_K^2 + m_K^2} + \sqrt{p_K^2 + m_\pi^2} = m_D$,

leading to $p_K = \left[\left(\frac{m_D^2 + m_\pi^2 - m_k^2}{2m_D}\right)^2 - m_\pi^2\right]^{1/2} = 0.82$ GeV/c.

Correct option is (a)

27. A pion of mass 140 MeV/c^2 decays at rest in the laboratory to a muon of mass 105 MeV/c^2 and massless neutrino. The momentum of the muon in the laboratory is
(a) 30.625 MeV/c (b) 15.313 MeV/c (c) 40.833 MeV/c (d) 20.147 MeV/c

Soln. Therefore, initial momentum = 0
So, total final momentum should be zero {according to law of conservation of momentum}
So, $|p|_{\text{muon}} = |p|_{\text{neutrino}} = p$
So, from law of conservation of energy.

$$140 = \sqrt{p^2c^2 + (105)^2} + pc$$

$$\Rightarrow \quad 140 - pc = \sqrt{p^2c^2 + (105)^2}$$

Squaring both sides, so

$$\Rightarrow \quad 140^2 + p^2c^2 - 280\,pc = p^2c^2 + (105)^2$$

$$280\,pc = 140^2 - 105^2$$

$$\Rightarrow \quad 280\,pc = 8575$$

$$pc = 30.625 \text{ MeV}$$

$$\Rightarrow \quad p = 30.625 \text{ MeV}/c$$

Correct option is (a)

CAREER ENDEAVOUR

PRACTICE SET

1. Consider the four processes

(i) $p^+ \to n + e^+ + \nu_e$ (ii) $\Lambda^0 \to p^+ + e^+ + \nu_e$

(iii) $\pi^+ \to e^+ + \nu_e$ (iv) $\pi^0 \to \gamma + \gamma$

Which of the above is/are forbidden for free particles?

(a) only (ii) (b) (ii) and (iv) (c) (i) and (iv) (d) (i) and (ii)

2. Consider the following ratios of the partial decay widths $R_1 = \frac{\Gamma(\rho^+ \to \pi^+ + \pi^0)}{\Gamma(\rho^- \to \pi^- + \pi^0)}$ and $R_2 = \frac{\Gamma(\Delta^{++} \to \pi^+ + p)}{\Gamma(\Delta^- \to \pi^- + n)}$. If the effects of electromagnetic and weak interactions are neglected, then R_1 and R_2 are, respectively,

(a) 1 and $\sqrt{2}$ (b) 1 and 2 (c) 2 and 1 (d) 1 and 1

3. Consider the following particles: the proton p, the neutron n, the neutral pion π^0 and the delta resonance Δ^+. When ordered in terms of decreasing lifetime, the correct arrangement is as follows:

(a) π^0, n, p, Δ^+ (b) p, n, Δ^+, π^0 (c) p, n, π^0, Δ^+ (d) Δ^+, n, π^0, p

4. A pion π^+ of rest mass 140 MeV/c^2 is accelerated to an energy of 2.5 GeV. This cannot decay to a proton of rest mass 938 MeV/c^2 and an anti-neutron of rest mass 939 MeV/c^2 because

(a) a moving pion cannot produce a proton at rest
(b) energy and momentum cannot be simultaneously conserved
(c) the spin of the pion is different from the spin of the proton
(d) it is forbidden by isospin conservation

5. Which of the following quantities of an elementary particle should be zero if parity is conserved?

(a) mass width (b) magnetic dipole moment
(c) electric quadrupole moment (d) electric dipole moment

6. The prinicipal decay mode of a neutral pion π^0 is through the process

(a) $\pi^0 \to \gamma\gamma$ (b) $\pi^0 \to e^+e^-$ (c) $\pi^0 \to \mu^+\mu^-$ (d) $\pi^0 \to \nu\bar{\nu}$

7. Which one of the following sets corresponds to fundamental particles?

(a) proton, electron and neutron (b) proton, electron and photon
(c) electron, photon and neutrino (d) quark, electron and meson

8. A delta baryon is found to have charge +2 and strangeness 0. Its isospin must be

(a) 2 (b) 3/2 (c) 1 (d) $\frac{1}{2}$

9. A muon participates is
(a) electromagnetic, weak and gravitational interactions only
(b) weak and gravitational interactions only
(c) weak and strong interaction only
(d) electromagnetic and gravitational interactions only

10. The reaction $p+\pi^- \rightarrow n$ is allowed by :

(a) strong inetraction (b) E.M. interaction (c) weak interaction (d) gravitational interaction.

ANSWER KEY					
Questions	1	2	3	4	5
Option	(d)	(d)	(c)	(b)	(d)
Questions	6	7	8	9	10
Option	(a)	(c)	(b)	(a)	(b)

CAREER ENDEAVOUR

REFERENCE BOOKS

1. Introductory Nuclear Physics : Kenneth S. Krane
2. Introductory Nuclear Physics : David Halliday
3. Introduction to Elementary Particles : David J. Griffiths
4. An Introduction to Nuclear Physics : W. N. Cottingham and D. A. Greenwood
5. Concepts of Modern Physics : Arthur Beiser
6. Concepts of Nuclear Physics: Bernard L. Cohen
7. Introduction to the Physics of Nuclei & Particles : R. A. Dunlap
8. Introduction to High Energy Physics : D. H. Perkins
9. Particle Physics : C. L. Arora